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IIB supergravity revisited

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Abstract: We show in the $\mathrm{SU}(1,1)$-covariant formulation that IIB supergravity allows the introduction of a doublet and a quadruplet of ten-form potentials. The Ramond-Ramond ten-form potential which is associated with the SO(32) type-I superstring is in the quadruplet. Our results are consistent with a recently proposed E_{11} symmetry underlying string theory.
For the reader's convenience we present the full supersymmetry and gauge transformations of *all* fields both in the manifestly $\mathrm{SU}(1,1)$ covariant Einstein frame and in the real U(1) gauge fixed string frame.

Keywords: Field Theories in Higher Dimensions, Extended Supersymmetry, Supergravity Models.

Contents

1. Introduction

IIB supergravity [1 − 3] is the low energy effective action of type-IIB superstring theory. Its scalar sector describes the coset manifold $\mathrm{SL}(2,\mathbb{R})/\mathrm{SO}(2) \simeq \mathrm{SU}(1,1)/\mathrm{U}(1)$, whose isometry $\mathrm{SL}(2,\mathbb{R})$ is a symmetry of the low energy theory. Since the isometry acts non-trivially on the dilaton, the full perturbative string theory does not preserve the symmetry, but the conjecture is that non-perturbatively an $\mathrm{SL}(2,\mathbb{Z})$ subgroup of the full symmetry group of the low energy action survives [4].

The particular feature of type-IIB string theory with respect to the other theories of closed oriented strings is that it is symmetric under the orientation reversal of the fundamental string. Ten-dimensional type-I string theory is obtained from type-IIB through an orientifold projection [5] that gauges this symmetry, and tadpole cancellation requires the introduction of an open sector, corresponding to D9-branes. The standard supersymmetric projection gives rise to the type-I superstring, with gauge group SO(32) [6], while a non-supersymmetric, anomaly-free projection gives rise to a model with gauge group USp(32) [7], in which supersymmetry is realized on the bulk and spontaneously broken on the branes [8].

In the low-energy effective action, the closed sector of type-I strings is obtained by performing a consistent $\mathbb{Z}_2$ truncation of the IIB supergravity, while the open sector corresponds to the first order in the low-energy expansion of the D9-brane action in a type-I background. In [9] it was shown that the $\mathbb{Z}_2$ symmetry responsible for this truncation can be performed in two ways, and in a flat background, with all bulk fields put to zero, the D9-brane action reduces in one case to the Volkov-Akulov action [10], and in the other case to a constant. In [11] these results were extended to a generic background, showing that also in the curved case there are two possibilities of performing the truncation. In one case one gets a dilaton tadpole and a RR tadpole plus goldstino couplings, which is basically the one-brane equivalent of the Sugimoto model, while in the other case the goldstino couplings vanish and one is left with a dilaton and a RR tadpole, which is the one-brane equivalent of the supersymmetric model. In order to truncate the theory in the brane sector, the "democratic formulation" of IIB supergravity was derived [9, 12]. This amounts to an extension of the supersymmetry algebra, so that both the RR fields and their magnetic duals appear on the same footing. The closure of the algebra then requires the field strengths of these fields to be related by duality conditions. The result is that, together with the RR forms $C^{(2n)}$, $n = 0, \ldots, 4$ associated with D-branes of non-vanishing codimension, the algebra naturally includes a RR ten-form $C^{(10)}$, with respect to which the spacetime-filling D9-branes are electrically charged. This field does not have any field strength, and correspondingly an object charged with respect to it can be consistently included in the theory only when one performs a type-I truncation, so that the resulting overall RR charge vanishes. The analysis of [9] also showed that an additional ten-form $B^{(10)}$ can be introduced in the algebra, and this form survives a different $\mathbb{Z}_2$ truncation, projecting out all the RR-fields. In the string frame, the tension of a spacetime-filling brane electrically charged with respect to $B^{(10)}$ would scale like g_S^{-2}, instead of g_S^{-4}, thus implying that the brane action for this object can not be obtained performing an S-duality transformation on the D9-brane effective action [13]. We are therefore facing a problem, since two ten-forms are known in IIB supergravity, but they do not form a doublet with respect to $\mathrm{SL}(2,\mathbb{R})$.

In this paper we will clarify this issue. We want to obtain all the possible independent ten-forms that can be added to 10-dimensional IIB supergravity, with their assignment to representations of $\mathrm{SL}(2,\mathbb{R})$. In order to perform this analysis, we express the theory in a "$\mathrm{SU}(1,1)$-democratic formulation", in which all the forms, not only the RR ones, and their magnetic duals are described in a $\mathrm{SU}(1,1)$-covariant way. We use the notation of [1, 2], so that the scalars parametrize the coset $\mathrm{SU}(1,1)/\,\mathrm{U}(1)$, while the two two-forms, as well as their duals, form a doublet of $\mathrm{SU}(1,1)$. The eight-forms, dual to the scalars, transform as a triplet of $\mathrm{SU}(1,1)$, with the field strengths satisfying an $\mathrm{SU}(1,1)$ invariant constraint [14, 15]. Eventually, we find that the algebra includes a doublet and a quadruplet of ten-forms,[1] and the dilaton dependence of the supersymmetry transformation of these objects shows that the RR ten-form belongs to the quadruplet. We claim that no other independent ten-forms can be added to the algebra. In summary, we find the following bosonic field content:

$$e_\mu^a, V_+^\alpha, V_-^\alpha, A_{(2)}^\alpha, A_{(4)}, A_{(6)}^\alpha, A_{(8)}^{(\alpha\beta)}, A_{(10)}^\alpha, A_{(10)}^{(\alpha\beta\gamma)}, \tag{1.1}$$

[1]Gauge fields of maximal rank have been explored in the literature [16].

where e_μ^a is the zehnbein, (V_+^α, V_-^α) parametrizes the $\mathrm{SU}(1,1)/\,\mathrm{U}(1)$ coset, $\alpha = 1,2$ is an $\mathrm{SU}(1,1)$ index and the subindex (n) indicates the rank of the potential.

This paper will be devoted to the construction and the properties of the extended IIB supergravity theory (1.1). Clearly the properties of the dual forms and ten-forms have implications for the structure of the brane spectrum, dualities, etc. These aspects of this work will be addressed in a forthcoming paper [17].

The structure of the paper is as follows. The main result, the supersymmetry transformation rules and algebra of the extended IIB-supergravity theory in the $\mathrm{SU}(1,1)/\,\mathrm{U}(1)$ formulation, are given in section 5. In section 6 these results are rewritten in a $\mathrm{U}(1)$ gauge in the Einstein frame and in the string frame. In this section we also recover the Ramond-Ramond "harmonica" of [9] and then extend it to the Neveu-Schwarz forms. We also list the action of S-duality on all form fields. The preceding sections lead up to these results and sketch the derivation. In section 2 we review the $\mathrm{SU}(1,1)$-covariant notation of [1, 2]. In section 3 we introduce in the algebra the six- and the eight-forms dual to the two-forms and the scalars respectively. Section 4 contains the analysis of the ten-forms. We finally conclude with a summary of our results and a discussion. Some basic formulas and truncations to $N = 1$ supergravity can be found in the appendices.

2. The $\mathrm{SU}(1,1)$-covariant formulation

In this section we review the notation and the results of [1, 2].

The theory contains the graviton, two scalars, two two-forms and a self-dual four-form in the bosonic sector, together with a complex left-handed gravitino and a complex right-handed spinor in the fermionic sector. We will use the mostly-minus spacetime signature convention throughout the paper. The two scalars parametrize the coset $\mathrm{SU}(1,1)/\,\mathrm{U}(1)$, that can be described in terms of the $\mathrm{SU}(1,1)$ matrix $(\alpha, \beta = 1,2)$

$$U = (V_-^\alpha \; V_+^\alpha)\,, \tag{2.1}$$

satisfying the constraint

$$V_-^\alpha V_+^\beta - V_+^\alpha V_-^\beta = \epsilon^{\alpha\beta}\,, \tag{2.2}$$

with $(V_-^1)^* = V_+^2$, where $\alpha = 1,2$ is an $\mathrm{SU}(1,1)$ index and $+$ and $-$ denote the $\mathrm{U}(1)$ charge, and $\epsilon^{12} = \epsilon_{12} = 1$. From the left-invariant 1-form

$$U^{-1}\partial_\mu U = \begin{pmatrix} -iQ_\mu & P_\mu \\ P_\mu^* & iQ_\mu \end{pmatrix} \tag{2.3}$$

one reads off the $\mathrm{U}(1)$-covariant quantity

$$P_\mu = -\epsilon_{\alpha\beta} V_+^\alpha \partial_\mu V_+^\beta\,, \tag{2.4}$$

that has charge 2, and the $\mathrm{U}(1)$ connection

$$Q_\mu = -i\epsilon_{\alpha\beta} V_-^\alpha \partial_\mu V_+^\beta\,. \tag{2.5}$$

Note that

$$P_\mu V_-^\alpha = D_\mu V_+^\alpha \,, \tag{2.6}$$

$$P_\mu^* V_+^\alpha = D_\mu V_-^\alpha \,, \tag{2.7}$$

where the derivative D is covariant with respect to U(1). The two-forms are collected in an SU(1,1) doublet $A^\alpha_{\mu\nu}$ satisfying the constraint

$$(A^1_{\mu\nu})^* = A^2_{\mu\nu} \,. \tag{2.8}$$

The corresponding field strengths

$$F^\alpha_{\mu\nu\rho} = 3\partial_{[\mu} A^\alpha_{\nu\rho]} \tag{2.9}$$

are invariant with respect to the gauge transformations

$$\delta A^\alpha_{\mu\nu} = 2\partial_{[\mu} \Lambda^\alpha_{\nu]} \,. \tag{2.10}$$

The four-form is invariant under SU(1,1), and varies as

$$\delta A_{\mu\nu\rho\sigma} = 4\partial_{[\mu} \Lambda_{\nu\rho\sigma]} - \frac{i}{4} \epsilon_{\alpha\beta} \Lambda^\alpha_{[\mu} F^\beta_{\nu\rho\sigma]} \tag{2.11}$$

under four-form and two-form gauge transformations, so that the gauge-invariant five-form field-strength is

$$F_{\mu\nu\rho\sigma\tau} = 5\partial_{[\mu} A_{\nu\rho\sigma\tau]} + \frac{5i}{8} \epsilon_{\alpha\beta} A^\alpha_{[\mu\nu} F^\beta_{\rho\sigma\tau]} \,. \tag{2.12}$$

This five-form satisfies the self-duality condition

$$F^{\mu_1 \ldots \mu_5} = \frac{1}{5!} \epsilon^{\mu_1 \ldots \mu_5 \nu_1 \ldots \nu_5} F_{\nu_1 \ldots \nu_5} \,. \tag{2.13}$$

It is convenient to define the complex three-form

$$G_{\mu\nu\rho} = -\epsilon_{\alpha\beta} V_+^\alpha F^\beta_{\mu\nu\rho} \,, \tag{2.14}$$

that is an SU(1,1) singlet with U(1) charge 1. Finally the gravitino ψ_μ is complex left-handed with U(1) charge 1/2, while the spinor λ is complex right-handed with U(1) charge 3/2.

In [2] the field equations for this model were derived by requiring the closure of the supersymmetry algebra. All these equations can be derived from a lagrangian, imposing eq. (2.13) only after varying [18].[2] It is interesting to study in detail the kinetic term for the scalar fields,

$$\mathcal{L}_{\text{scalar}} = \frac{e}{2} P_\mu^* P^\mu \,. \tag{2.15}$$

[2]A lagrangian formulation for self dual forms has been developed in [19], and then applied in [15] to the ten-dimensional IIB supergravity. It corresponds to the introduction of an additional scalar auxiliary field, and the self-duality condition results from the gauge fixing (that can not be imposed directly on the action) of additional local symmetries.

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The complex variable

$$z = \frac{V_-^2}{V_-^1} \tag{2.16}$$

is invariant under local U(1) transformations, and so it is a good coordinate for the scalar manifold. Under the SU(1,1) transformation

$$\begin{pmatrix} V_-^1 \\ V_-^2 \end{pmatrix} \to \begin{pmatrix} \alpha & \beta \\ \bar{\beta} & \bar{\alpha} \end{pmatrix} \begin{pmatrix} V_-^1 \\ V_-^2 \end{pmatrix} , \tag{2.17}$$

that is an isometry of the scalar manifold, z transforms as

$$z \to \frac{\bar{\alpha} z + \bar{\beta}}{\beta z + \alpha} . \tag{2.18}$$

The variable z parametrizes the unit disc, $|z| < 1$, and the kinetic term assumes the form

$$\mathcal{L}_{\text{scalar}} = -\frac{e}{2} \frac{\partial_\mu z \partial^\mu \bar{z}}{(1 - z\bar{z})^2} . \tag{2.19}$$

The further change of variables

$$z = \frac{1 + i\tau}{1 - i\tau} \tag{2.20}$$

maps the disc in the complex upper-half plane, $\text{Im}\tau > 0$, and in terms of τ the transformations (2.17) become

$$\tau \to \frac{a\tau + b}{c\tau + d} , \tag{2.21}$$

where

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \text{SL}(2, \mathbb{R}) , \tag{2.22}$$

while the scalar lagrangian takes the form

$$\mathcal{L}_{\text{scalar}} = -\frac{e}{8} \frac{\partial_\mu \tau \partial^\mu \bar{\tau}}{(\text{Im}\tau)^2} . \tag{2.23}$$

Expressing τ in terms of the RR scalar and the dilaton,

$$\tau = \ell + i e^{-\phi} \tag{2.24}$$

and performing the Weyl rescaling $g_{(E)\mu\nu} \to e^{-\phi/2} g_{(S)\mu\nu}$ one ends up with the standard form of the kinetic term of the scalars in IIB supergravity in the string frame.

The supersymmetry transformations that leave the field equations of [2] invariant are

$$\begin{aligned}
\delta e_\mu{}^a &= i\bar{\epsilon}\gamma^a \psi_\mu + i\bar{\epsilon}_C \gamma^a \psi_{\mu C} , \\
\delta \psi_\mu &= D_\mu \epsilon + \frac{i}{480} F_{\mu\nu_1 \ldots \nu_4} \gamma^{\nu_1 \ldots \nu_4} \epsilon + \frac{1}{96} G^{\nu\rho\sigma} \gamma_{\mu\nu\rho\sigma} \epsilon_C - \frac{3}{32} G_{\mu\nu\rho} \gamma^{\nu\rho} \epsilon_C , \\
\delta A_{\mu\nu}^\alpha &= V_-^\alpha \; \bar{\epsilon}\gamma_{\mu\nu} \lambda + V_+^\alpha \; \bar{\epsilon}_C \gamma_{\mu\nu} \lambda_C + 4iV_-^\alpha \; \bar{\epsilon}_C \gamma_{[\mu} \psi_{\nu]} + 4iV_+^\alpha \; \bar{\epsilon}\gamma_{[\mu} \psi_{\nu]C} , \\
\delta A_{\mu\nu\rho\sigma} &= \bar{\epsilon}\gamma_{[\mu\nu\rho} \psi_{\sigma]} - \bar{\epsilon}_C \gamma_{[\mu\nu\rho} \psi_{\sigma]C} - \frac{3i}{8} \epsilon_{\alpha\beta} A_{[\mu\nu}^\alpha \delta A_{\rho\sigma]}^\beta ,
\end{aligned}$$

$$
\begin{aligned}
\delta\lambda &= iP_\mu\gamma^\mu\epsilon_C - \frac{i}{24}G_{\mu\nu\rho}\gamma^{\mu\nu\rho}\epsilon\,,\\
\delta V_+^\alpha &= V_-^\alpha\ \bar{\epsilon}_C\lambda\,,\\
\delta V_-^\alpha &= V_+^\alpha\ \bar{\epsilon}\lambda_C\,.
\end{aligned}
\tag{2.25}
$$

where we denote with Ψ_C the complex (Majorana) conjugate of Ψ. The commutator $[\delta_1, \delta_2]$ of two supersymmetry transformations of (2.25) closes on all the local symmetries of the theory, provided one uses the fermionic field equations and the self-duality condition of eq. (2.13). To lowest order in the fermions, the parameters of the resulting general coordinate transformation, four-form gauge transformation and two-form gauge transformation are[3]

$$
\begin{aligned}
\xi^\mu &= i\,\bar{\epsilon}_2\gamma^\mu\epsilon_1 + i\,\bar{\epsilon}_{2C}\gamma^\mu\epsilon_{1C}\,,\\
\Lambda_\mu^\alpha &= A_{\mu\nu}^\alpha\xi^\nu - 2i[V_+^\alpha\,\bar{\epsilon}_2\gamma_\mu\epsilon_{1C} + V_-^\alpha\,\bar{\epsilon}_{2C}\gamma_\mu\epsilon_1]\,,\\
\Lambda_{\mu\nu\rho} &= A_{\mu\nu\rho\sigma}\xi^\sigma - \frac{1}{4}[\bar{\epsilon}_2\gamma_{\mu\nu\rho}\epsilon_1 - \bar{\epsilon}_{2C}\gamma_{\mu\nu\rho}\epsilon_{1C}] -\\
&\quad -\frac{3}{8}\epsilon_{\alpha\beta}A_{[\mu\nu}^\alpha\big(V_+^\beta\,\bar{\epsilon}_2\gamma_{\rho]}\epsilon_{1C} + V_-^\beta\,\bar{\epsilon}_{2C}\gamma_{\rho]}\epsilon_1\big)\,.
\end{aligned}
\tag{2.26}
$$

In the next section we will extend the algebra in order to include the magnetic duals of the scalars and of the two-form, in such a way that the supersymmetry algebra still closes, once the proper duality relations are used. Once we obtain the supersymmetry transformation of the six- and the eight-forms that are compatible with the algebra obtained from eq. (2.25), we will include in section 4 all the possible independent ten-forms that this algebra allows.

3. Six-forms and eight-forms

In this section we show how the algebra of eq. (2.25) is extended introducing the forms magnetically dual to the scalars and the two-forms. As anticipated, closure of the supersymmetry algebra requires the field strengths of these forms to be related to P_μ and the field strengths of the two-forms by suitable duality relations. Generalizing what happens for the four-form (see eqs. (2.11) and (2.12)), we will see that the gauge transformations of these fields involve the gauge parameters of all the lower rank forms, and the gauge invariant field strengths will therefore contain lower rank forms as well. After introducing our Ansatz for these field strengths and gauge transformations, the supersymmetry transformations of these fields will then be determined requiring the closure of the supersymmetry algebra. As in the previous section, we will not consider terms higher than quadratic in the fermi fields.

3.1 Six-forms

We want to obtain the gauge and supersymmetry transformations for the doublet of six-forms $A^\alpha_{\mu_1\ldots\mu_6}$, which are the magnetic duals of the two-forms and thus satisfy the reality

[3]We only present the parameters of translations and the two- and four-form gaugetransformations. The parameters of other local symmetries, namely supersymmetry, local Lorentz and local U(1) are not used in the analysis of the next sections, and are given in [1, 2].

condition

$$(A^1)^*_{\mu_1\ldots\mu_6} = A^2_{\mu_1\ldots\mu_6}\,. \tag{3.1}$$

Generalizing what one obtains for the four-form, we expect the supersymmetry transformation of the six-forms to contain terms involving only spinors and terms containing forms of lower rank. The condition of eq. (3.1), as well as the requirement that all the terms must have vanishing local U(1) charge, fixes the most general transformation of the doublet to be

$$\begin{aligned}\delta A^\alpha_{\mu_1\cdots\mu_6} &= a\, V^\alpha_-\,\bar\epsilon\gamma_{\mu_1\ldots\mu_6}\lambda + a^*\, V^\alpha_+\,\bar\epsilon_C\gamma_{\mu_1\ldots\mu_6}\lambda_C + b\, V^\alpha_-\,\bar\epsilon_C\gamma_{[\mu_1\ldots\mu_5}\psi_{\mu_6]} - \\ &\quad -b^*\, V^\alpha_+\,\bar\epsilon\gamma_{[\mu_1\ldots\mu_5}\psi_{\mu_6]C} + c\, A_{[\mu_1\cdots\mu_4}\delta A^\alpha_{\mu_5\mu_6]} + d\,\delta A_{[\mu_1\cdots\mu_4}A^\alpha_{\mu_5\mu_6]} + \\ &\quad +ie\,\epsilon_{\beta\gamma}\delta A^\beta_{[\mu_1\mu_2}A^\gamma_{\mu_3\mu_4}A^\alpha_{\mu_5\mu_6]}\,. \end{aligned} \tag{3.2}$$

We want to consider the commutator $[\delta_1,\delta_2]$ of two such transformations, to lowest order in the fermi fields.

We first take into account the terms involving the spinors, i.e., the first two lines in eq. (3.2). Those terms produce the gauge transformation for the six-forms

$$\begin{aligned}\delta A^\alpha_{\mu_1\ldots\mu_6} &= 6\partial_{[\mu_1}\Lambda^\alpha_{\mu_2\ldots\mu_6]} \\ &= -12i\partial_{[\mu_1}(a\ V^\alpha_+\ \bar\epsilon_2\gamma_{\mu_2\ldots\mu_6]}\epsilon_{1C} + a^*\ V^\alpha_-\ \bar\epsilon_{2C}\gamma_{\mu_2\ldots\mu_6]}\epsilon_1)\end{aligned} \tag{3.3}$$

if the constraint

$$12ia^* = b \tag{3.4}$$

is imposed, while the other terms that are produced are

$$20iaF^\alpha_{[\mu_1\mu_2\mu_3}(\bar\epsilon_{2C}\gamma_{\mu_4\mu_5\mu_6]}\epsilon_{1C} - \bar\epsilon_2\gamma_{\mu_4\mu_5\mu_6]}\epsilon_1) - \frac{1}{6}a\epsilon_{\mu_1\ldots\mu_6\sigma\mu\nu\rho}S^{\alpha\beta}\epsilon_{\beta\gamma}F^{\gamma;\mu\nu\rho}\xi^\sigma\,, \tag{3.5}$$

where we have defined

$$S^{\alpha\beta} = V^\alpha_- V^\beta_+ + V^\alpha_+ V^\beta_- \tag{3.6}$$

and we have assumed that a is imaginary. Note that $S^{\alpha\beta}$ satisfies

$$S^{\alpha\beta}\epsilon_{\beta\gamma}S^{\gamma\delta}\epsilon_{\delta\epsilon} = \delta^\alpha_\epsilon\,. \tag{3.7}$$

Observe that there are no terms involving the five-form field strength. Without loss of generality, we fix

$$a = i \tag{3.8}$$

from now on. In order for the last term in (3.5) to produce a general coordinate transformation with the right coefficient as dictated by eq. (2.26), we impose the duality relation[4]

$$F^\alpha_{\mu_1\ldots\mu_7} = -\frac{i}{3!}\epsilon_{\mu_1\ldots\mu_7\mu\nu\rho}S^{\alpha\beta}\epsilon_{\beta\gamma}F^{\gamma;\mu\nu\rho}\,, \tag{3.9}$$

where $F^\alpha_{\mu_1\ldots\mu_7} = 7\partial_{[\mu_1}A^\alpha_{\mu_2\ldots\mu_7]} + \ldots$ are the field strengths of the six-forms, and the dots stand for terms involving lower rank forms that we will determine in the following. Note

[4]Note that this duality relation induces field equations for the potentials.

that the second term of eq. (3.5) contains, together with a general coordinate transformation, a gauge transformation of parameter

$$\Lambda'^{\alpha}_{\mu_1\ldots\mu_5} = A^{\alpha}_{\mu_1\ldots\mu_5\sigma}\xi^{\sigma}\,. \tag{3.10}$$

The SU(1, 1)-invariant quantities

$$G_{\mu_1\ldots\mu_7} = -\epsilon_{\alpha\beta}V^{\alpha}_{+}F^{\beta}_{\mu_1\ldots\mu_7}\,, \qquad G^{*}_{\mu_1\ldots\mu_7} = \epsilon_{\alpha\beta}V^{\alpha}_{-}F^{\beta}_{\mu_1\ldots\mu_7}\,, \tag{3.11}$$

which have U(1) charge +1 and −1 respectively, satisfy

$$G^{(7)}_{\mu_1\ldots\mu_7} = \frac{i}{3!}\epsilon_{\mu_1\ldots\mu_7\mu\nu\rho}G^{\mu\nu\rho}\,, \qquad G^{*}_{\mu_1\ldots\mu_7} = -\frac{i}{3!}\epsilon_{\mu_1\ldots\mu_7\mu\nu\rho}G^{*\,\mu\nu\rho}\,. \tag{3.12}$$

In order to proceed further, in analogy with eq. (2.12) we make the following Ansatz for the seven-form field strengths:

$$F^{\alpha}_{\mu_1\ldots\mu_7} = 7\partial_{[\mu_1}A^{\alpha}_{\mu_2\ldots\mu_7]} + \alpha A^{\alpha}_{[\mu_1\mu_2}F_{\mu_3\ldots\mu_7]} + \beta F^{\alpha}_{[\mu_1\ldots\mu_3}A_{\mu_4\ldots\mu_7]}\,. \tag{3.13}$$

For these forms to be gauge invariant, the must transform non-trivially with respect to the two-form and four-form gauge transformations. The result is

$$\delta A^{\alpha}_{\mu_1\ldots\mu_6} = -\frac{2}{7}\alpha\Lambda^{\alpha}_{[\mu_1}F_{\mu_2\ldots\mu_6]} + \frac{4}{7}\beta F^{\alpha}_{[\mu_1\ldots\mu_3}\Lambda_{\mu_4\ldots\mu_6]}\,, \tag{3.14}$$

and gauge invariance requires

$$\beta = -\frac{10}{3}\alpha\,. \tag{3.15}$$

Now we come back to the commutator. The terms that are left are the ones coming from the last three lines in eq. (3.2), together with the first line in eq. (3.5) and the terms coming from (3.13) in the second line of eq. (3.5). All these terms have to produce gauge transformations according to (3.14), with parameters given from eqs. (2.26), possibly together with additional gauge transformations. The end result is that one produces the additional gauge transformations

$$\begin{aligned}\Lambda''^{\alpha}_{\mu_1\ldots\mu_5} = &-\frac{2i}{3}c\ A_{[\mu_1\ldots\mu_4}(V^{\alpha}_{+}\ \bar{\epsilon}_2\gamma_{\mu_5]}\epsilon_{1C} + V^{\alpha}_{-}\ \bar{\epsilon}_{2C}\gamma_{\mu_5]}\epsilon_1) - \\ &-\frac{1}{6}d\ A^{\alpha}_{[\mu_1\mu_2}(\bar{\epsilon}_2\gamma_{\mu_3\ldots\mu_5]}\epsilon_1 - \bar{\epsilon}_{2C}\gamma_{\mu_3\ldots\mu_5]}\epsilon_{1C})\,,\end{aligned} \tag{3.16}$$

while all the coefficients are uniquely determined to be

$$c = 40\,, \qquad d = -20\,, \qquad e = \frac{15}{2}\,, \qquad \alpha = 28\,. \tag{3.17}$$

Summarizing, we get that the supersymmetry transformations of the six-forms are

$$\begin{aligned}\delta A^{\alpha}_{\mu_1\cdots\mu_6} = &\ i\ V^{\alpha}_{-}\ \bar{\epsilon}\gamma_{\mu_1\ldots\mu_6}\lambda - i\ V^{\alpha}_{+}\ \bar{\epsilon}_C\gamma_{\mu_1\ldots\mu_6}\lambda_C + 12\ V^{\alpha}_{-}\ \bar{\epsilon}_C\gamma_{[\mu_1\ldots\mu_5}\psi_{\mu_6]} - \\ &-12\ V^{\alpha}_{+}\ \bar{\epsilon}\gamma_{[\mu_1\ldots\mu_5}\psi_{\mu_6]C} + 40\ A_{[\mu_1\cdots\mu_4}\delta A^{\alpha}_{\mu_5\mu_6]} \\ &-20\ \delta A_{[\mu_1\cdots\mu_4}A^{\alpha}_{\mu_5\mu_6]} + \frac{15i}{2}\epsilon_{\beta\gamma}\delta A^{\beta}_{[\mu_1\mu_2}A^{\gamma}_{\mu_3\mu_4}A^{\alpha}_{\mu_5\mu_6]}\,.\end{aligned} \tag{3.18}$$

The doublet of seven-form field strengths is

$$F^\alpha_{\mu_1\dots\mu_7} = 7\partial_{[\mu_1} A^\alpha_{\mu_2\dots\mu_7]} + 28 A^\alpha_{[\mu_1\mu_2} F_{\mu_3\dots\mu_7]} - \frac{280}{3} F^\alpha_{[\mu_1\dots\mu_3} A_{\mu_4\dots\mu_7]} \,. \tag{3.19}$$

This is gauge invariant with respect to the transformations of the two-forms, the four-form and the six-forms, where

$$\delta A^\alpha_{\mu_1\dots\mu_6} = 6\partial_{[\mu_1} \Lambda^\alpha_{\mu_2\dots\mu_6]} - 8\Lambda^\alpha_{[\mu_1} F_{\mu_2\dots\mu_6]} - \frac{160}{3} F^\alpha_{[\mu_1\dots\mu_3} \Lambda_{\mu_4\dots\mu_6]} \,. \tag{3.20}$$

Moreover, the six-form gauge transformation parameter resulting from the commutator of two supersymmetry transformations is

$$\begin{aligned}\Lambda^\alpha_{\mu_1\dots\mu_5} = {} & A^\alpha_{\mu_1\dots\mu_5\sigma}\xi^\sigma + 2(V^\alpha_+ \, \bar\epsilon_2 \gamma_{\mu_1\dots\mu_5}\epsilon_1 - V^\alpha_- \, \bar\epsilon_{2C}\gamma_{\mu_1\dots\mu_5}\epsilon_{1C}) - \\ & - \frac{80i}{3} A_{[\mu_1\dots\mu_4}(V^\alpha_+ \, \bar\epsilon_2\gamma_{\mu_5]}\epsilon_{1C} + V^\alpha_- \, \bar\epsilon_{2C}\gamma_{\mu_5]}\epsilon_1) + \\ & + \frac{10}{3} A^\alpha_{[\mu_1\mu_2}(\bar\epsilon_2\gamma_{\mu_3\dots\mu_5]}\epsilon_1 - \bar\epsilon_{2C}\gamma_{\mu_3\dots\mu_5]}\epsilon_{1C}) \,,\end{aligned} \tag{3.21}$$

as results from eqs. (3.3), (3.10) and (3.16). Finally, a comment is in order. At first sight, the Ansatz we made for the field strengths in eq. (3.13) does not seem to be the most general one, since one could in principle include a term of the form $i\epsilon_{\beta\gamma} A^a_{[\mu_1\mu_2} A^\beta_{\mu_3\mu_4} F^\gamma_{\mu_5\dots\mu_7]}$. The reason why we did not include it is that one can always reabsorb such a term by performing a redefinition of the six-forms of the type $A^\alpha_{\mu_1\dots\mu_6} \to A^\alpha_{\mu_1\dots\mu_6} + \gamma A^\alpha_{[\mu_1\mu_2} A_{\mu_3\dots\mu_6]}$, and choose γ so that this term vanishes. This freedom will be used to constrain the form of the field strengths of the eight-forms as well, as we will see in the next subsection.

3.2 Eight-forms

The eight-forms are the magnetic duals of the scalars. As we reviewed in section 2, the scalars are described in terms of the left-invariant 1-form of eq. (2.3), transforming in the adjoint of SU(1, 1), and propagating two real degrees of freedom because of local U(1) invariance. One therefore expects a triplet of eight-forms (as observed in [14, 15]),[5] that we denote by $A^{\alpha\beta}_{\mu_1\dots\mu_8}$, symmetric under $\alpha \leftrightarrow \beta$, and satisfying the reality condition

$$(A^{11})^*_{\mu_1\dots\mu_8} = A^{22}_{\mu_1\dots\mu_8} \,, \qquad (A^{12})^*_{\mu_1\dots\mu_8} = A^{12}_{\mu_1\dots\mu_8} \,. \tag{3.22}$$

The fact that only two scalars propagate will result in a constraint for the field strengths of these eight-forms [20, 15]. This is exactly what we are going to show in this subsection. Following the same arguments as in the previous subsection, we write the most general supersymmetry transformations for the eight-forms, compatible with the reality condition and with U(1) invariance, consisting of terms that only involve the spinors and terms containing the lower rank forms and their supersymmetry transformations. The result is

$$\begin{aligned}\delta A^{\alpha\beta}_{\mu_1\dots\mu_8} = {} & a\, V^\alpha_+ V^\beta_+ \, \bar\epsilon\gamma_{\mu_1\dots\mu_8}\lambda_C + a^* \, V^\alpha_- V^\beta_- \, \bar\epsilon_C\gamma_{\mu_1\dots\mu_8}\lambda + \\ & + b\, V^{(\alpha}_+ V^{\beta)}_- \, \bar\epsilon\gamma_{[\mu_1\dots\mu_7}\psi_{\mu_8]} - b^* \, V^{(\alpha}_+ V^{\beta)}_- \, \bar\epsilon_C\gamma_{[\mu_1\dots\mu_7}\psi_{\mu_8]C} +\end{aligned}$$

[5] A similar observation was made for the curvatures in [20].

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$$+cA^{(\alpha}_{[\mu_1\dots\mu_6}\delta A^{\beta)}_{\mu_7\mu_8]}+dA^{(\alpha}_{[\mu_1\mu_2}\delta A^{\beta)}_{\mu_3\dots\mu_8]}+ieA^{(\alpha}_{[\mu_1\mu_2}A^{\beta)}_{\mu_3\mu_4}\epsilon_{\gamma\delta}A^{\gamma}_{\mu_5\mu_6}\delta A^{\delta}_{\mu_7\mu_8]}+$$
$$+fA^{(\alpha}_{[\mu_1\mu_2}A^{\beta)}_{\mu_3\mu_4}\delta A_{\mu_5\dots\mu_8]}+gA_{[\mu_1\dots\mu_4}A^{(\alpha}_{\mu_5\mu_6}\delta A^{\beta)}_{\mu_7\mu_8]}\,. \tag{3.23}$$

We first consider the contributions coming from the first two lines of eq. (3.23), in order to get a relation between a and b. We obtain the gauge transformation

$$\begin{aligned}\delta A^{\alpha\beta}_{\mu_1\dots\mu_8} &= 8\partial_{[\mu_1}\Lambda^{\alpha\beta}_{\mu_2\dots\mu_8]}\\ &= -4ia\partial_{[\mu_1}\left[S^{\alpha\beta}(\bar\epsilon_2\gamma_{\mu_2\dots\mu_8]}\epsilon_1-\bar\epsilon_{2C}\gamma_{\mu_2\dots\mu_8]}\epsilon_{1C})\right]\end{aligned} \tag{3.24}$$

together with the terms

$$\begin{aligned}&28ia(V_+^{(\alpha}\ \bar\epsilon_2\gamma_{[\mu_1\dots\mu_5}\epsilon_1-V_-^{(\alpha}\ \bar\epsilon_{2C}\gamma_{[\mu_1\dots\mu_5}\epsilon_{1C})F^{\beta)}_{\mu_6\dots\mu_8]}-\\ &-4a(V_+^{(\alpha}\ \bar\epsilon_2\gamma_{[\mu_1}\epsilon_1+V_-^{(\alpha}\ \bar\epsilon_{2C}\gamma_{[\mu_1}\epsilon_{1C})F^{\beta)}_{\mu_2\dots\mu_8]}-\\ &-a\epsilon_{\mu_1\dots\mu_8\sigma\tau}\xi^\sigma(V_+^\alpha V_+^\beta P^{*\tau}-V_-^\alpha V_-^\beta P^\tau)\,,\end{aligned} \tag{3.25}$$

provided that

$$8ia=b \tag{3.26}$$

and a is chosen to be imaginary. Fixing, without loss of generality,

$$a=-i\,, \tag{3.27}$$

one finds that the last term in eq. (3.25) contains the correct general coordinate transformation, plus an gauge transformation of parameter

$$\Lambda'^{\alpha\beta}_{\mu_1\dots\mu_7}=A^{\alpha\beta}_{\mu_1\dots\mu_7\sigma}\xi^\sigma \tag{3.28}$$

provided the duality relation

$$F^{\alpha\beta}_{\mu_1\dots\mu_9}=i\epsilon_{\mu_1\dots\mu_9}{}^{\sigma}[V_+^\alpha V_+^\beta P^*_\sigma-V_-^\alpha V_-^\beta P_\sigma] \tag{3.29}$$

holds, where $F^{\alpha\beta}_{\mu_1\dots\mu_9}=9\partial_{[\mu_1}A^{\alpha\beta}_{\mu_2\dots\mu_9]}+\dots$, and the dots stand for terms involving lower rank forms. From the field strengths of the eight-forms, one can define the SU$(1,1)$ invariant quantity

$$G_{\mu_1\dots\mu_9}=\epsilon_{\alpha\gamma}\epsilon_{\beta\delta}V_+^\alpha V_+^\beta F^{\gamma\delta}_{\mu_1\dots\mu_9}\,, \tag{3.30}$$

with U(1) charge +2, and its complex conjugate

$$G^*_{\mu_1\dots\mu_9}=\epsilon_{\alpha\gamma}\epsilon_{\beta\delta}V_-^\alpha V_-^\beta F^{\gamma\delta}_{\mu_1\dots\mu_9}\,. \tag{3.31}$$

In terms of these objects, the duality relation of eq. (3.29) becomes

$$G_{\mu_1\dots\mu_9}=-i\epsilon_{\mu_1\dots\mu_9\sigma}P^\sigma\,,\qquad G^*_{\mu_1\dots\mu_9}=i\epsilon_{\mu_1\dots\mu_9\sigma}P^{*\sigma}\,. \tag{3.32}$$

One can define a third nine-form,

$$\tilde{G}_{\mu_1 \ldots \mu_9} = \epsilon_{\alpha\gamma}\epsilon_{\beta\delta} V_+^\alpha V_-^\beta F^{\gamma\delta}_{\mu_1 \ldots \mu_9} \,, \tag{3.33}$$

with vanishing U(1) charge, but the duality relation (3.29) implies that this nine-form vanishes identically [15], thus determining an SU(1, 1) invariant constraint. Therefore only two eight-forms are actually independent.

We now come to our choice for the field strengths, for which the most general general expression is

$$\begin{aligned} F^{\alpha\beta}_{\mu_1 \ldots \mu_9} = \; & 9\partial_{[\mu_1} A^{\alpha\beta}_{\mu_2 \ldots \mu_9]} + \alpha F^{(\alpha}_{[\mu_1 \ldots \mu_7} A^{\beta)}_{\mu_8\mu_9]} + \beta F^{(\alpha}_{[\mu_1 \ldots \mu_3} A^{\beta)}_{\mu_4 \ldots \mu_9]} + \gamma F_{[\mu_1 \ldots \mu_5} A^{(\alpha}_{\mu_6\mu_7} A^{\beta)}_{\mu_8\mu_9]} + \\ & + i\delta\epsilon_{\gamma\delta} A^\gamma_{[\mu_1\mu_2} F^\delta_{\mu_3 \ldots \mu_5} A^{(\alpha}_{\mu_6\mu_7} A^{\beta)}_{\mu_8\mu_9]} + \xi A_{[\mu_1 \ldots \mu_4} F^{(\alpha}_{\mu_5 \ldots \mu_7} A^{\beta)}_{\mu_8\mu_9]} \,. \end{aligned} \tag{3.34}$$

The freedom of redefining the eight-form, $A_8 \to A_8 + A_6 A_2 + A_4 A_2 A_2$, can be used to put to zero the coefficients ξ and δ in (3.34). It turns out that defining the gauge transformation of the eight-forms as

$$\delta A^{\alpha\beta}_{\mu_1 \ldots \mu_8} = 8\partial_{[\mu_1} \Lambda^{\alpha\beta}_{\mu_2 \ldots \mu_8]} + \frac{2}{9}\alpha F^{(\alpha}_{[\mu_1 \ldots \mu_7} \Lambda^{\beta)}_{\mu_8]} + \frac{2}{3}\beta F^{(\alpha}_{[\mu_1 \ldots \mu_3} \Lambda^{\beta)}_{\mu_4 \ldots \mu_8]} \,, \tag{3.35}$$

the field strengths of eq. (3.34) are gauge invariant if the coefficient γ vanishes as well, and if the coefficients α and β are related by

$$\beta = -7\alpha \,. \tag{3.36}$$

To summarize, we have obtained

$$F^{\alpha\beta}_{\mu_1 \ldots \mu_9} = 9\partial_{[\mu_1} A^{\alpha\beta}_{\mu_2 \ldots \mu_9]} + \alpha F^{(\alpha}_{[\mu_1 \ldots \mu_7} A^{\beta)}_{\mu_8\mu_9]} - 7\alpha F^{(\alpha}_{\mu_1 \ldots \mu_3} A^{\beta)}_{\mu_4 \ldots \mu_9]} \,, \tag{3.37}$$

$$\delta A^{\alpha\beta}_{\mu_1 \ldots \mu_8} = 8\partial_{[\mu_1} \Lambda^{\alpha\beta}_{\mu_2 \ldots \mu_8]} + \frac{2}{9}\alpha F^{(\alpha}_{[\mu_1 \ldots \mu_7} \Lambda^{\beta)}_{\mu_8]} - \frac{14}{3}\alpha F^{(\alpha}_{[\mu_1 \ldots \mu_3} \Lambda^{\beta)}_{\mu_4 \ldots \mu_8]} \,. \tag{3.38}$$

We now consider the terms in the commutator coming from the last two lines of eq. (3.23), as well as the first two terms in eq. (3.25) and the part of the third containing lower rank forms. All these terms have to produce the gauge transformations of eq. (3.38) with the parameters given in eqs. (2.26) and (3.21), plus possibly a gauge transformation. The end result is that one produces the additional gauge transformation

$$\begin{aligned} \Lambda''^{\alpha\beta}_{\mu_1 \ldots \mu_7} = & -4ic \left[A^{(\alpha}_{[\mu_1 \ldots \mu_6} (V_+^{\beta)} \, \bar{\epsilon}_2 \gamma_{\mu_7]} \epsilon_1 + V_-^{\beta)} \, \bar{\epsilon}_{2C} \gamma_{\mu_7]} \epsilon_{1C}) \right] + \\ & + 12d \, \left[A^{(\alpha}_{[\mu_2\mu_3} (V_+^{\beta)} \, \bar{\epsilon}_2 \gamma_{\mu_4 \ldots \mu_7]} \epsilon_1 - V_-^{\beta)} \, \bar{\epsilon}_{2C} \gamma_{\mu_4 \ldots \mu_7]} \epsilon_{1C}) \right] , \end{aligned} \tag{3.39}$$

and the algebra closes provided that the coefficients are fixed to be

$$\begin{aligned} & c = \frac{21}{4} \,, \qquad d = -\frac{7}{4} \,, \qquad e = -\frac{105}{8} \,, \\ & f = -35 \,, \qquad g = 70 \,, \qquad \alpha = \frac{9}{4} \,. \end{aligned} \tag{3.40}$$

In conclusion, the supersymmetry transformation for the eight-forms is

$$\begin{aligned}\delta A^{\alpha\beta}_{\mu_1\ldots\mu_8} =& -i\ V_+^\alpha V_+^\beta\ \bar\epsilon\gamma_{\mu_1\ldots\mu_8}\lambda_C + i\ V_-^\alpha V_-^\beta\ \bar\epsilon_C\gamma_{\mu_1\ldots\mu_8}\lambda + \\ &+8\ V_+^{(\alpha}V_-^{\beta)}\ \bar\epsilon\gamma_{[\mu_1\ldots\mu_7}\psi_{\mu_8]} - 8\ V_+^{(\alpha}V_-^{\beta)}\ \bar\epsilon_C\gamma_{[\mu_1\ldots\mu_7}\psi_{\mu_8]C} + \\ &+\frac{21}{4}A^{(\alpha}_{[\mu_1\ldots\mu_6}\delta A^{\beta)}_{\mu_7\mu_8]} - \frac{7}{4}A^{(\alpha}_{[\mu_1\mu_2}\delta A^{\beta)}_{\mu_3\ldots\mu_8]} - \frac{105i}{8}A^{(\alpha}_{[\mu_1\mu_2}A^{\beta)}_{\mu_3\mu_4}\epsilon_{\gamma\delta}A^\gamma_{\mu_5\mu_6}\delta A^\delta_{\mu_7\mu_8]} - \\ &-35A^{(\alpha}_{[\mu_1\mu_2}A^{\beta)}_{\mu_3\mu_4}\delta A_{\mu_5\ldots\mu_8]} + 70A_{[\mu_1\ldots\mu_4}A^{(\alpha}_{\mu_5\mu_6}\delta A^{\beta)}_{\mu_7\mu_8]}\,, \end{aligned} \tag{3.41}$$

while the gauge invariance of the field strengths

$$F^{\alpha\beta}_{\mu_1\ldots\mu_9} = 9\partial_{[\mu_1}A^{\alpha\beta}_{\mu_2\ldots\mu_9]} + \frac{9}{4}F^{(\alpha}_{[\mu_1\ldots\mu_7}A^{\beta)}_{\mu_8\mu_9]} - \frac{63}{4}F^{(\alpha}_{[\mu_1\ldots\mu_3}A^{\beta)}_{\mu_4\ldots\mu_9]} \tag{3.42}$$

requires

$$\delta A^{\alpha\beta}_{\mu_1\ldots\mu_8} = 8\partial_{[\mu_1}\Lambda^{\alpha\beta}_{\mu_2\ldots\mu_8]} + \frac{1}{2}F^{(\alpha}_{[\mu_1\ldots\mu_7}\Lambda^{\beta)}_{\mu_8]} - \frac{21}{2}F^{(\alpha}_{[\mu_1\ldots\mu_3}\Lambda^{\beta)}_{\mu_4\ldots\mu_8]}\,. \tag{3.43}$$

Finally, the seven-form gauge parameter that appears in the commutator is

$$\begin{aligned}\Lambda^{\alpha\beta}_{\mu_1\ldots\mu_7} =& A^{\alpha\beta}_{\mu_1\ldots\mu_7\sigma}\xi^\sigma - \frac{1}{2}S^{\alpha\beta}[\bar\epsilon_2\gamma_{\mu_1\ldots\mu_7}\epsilon_1 - \bar\epsilon_{2C}\gamma_{\mu_1\ldots\mu_7}\epsilon_{1C}] - \\ &-\frac{21i}{8}A^\alpha_{[\mu_1\ldots\mu_6}(V_+^\beta\ \bar\epsilon_2\gamma_{\mu_7]}\epsilon_1 + V_-^\beta\ \bar\epsilon_{2C}\gamma_{\mu_7]}\epsilon_{1C}) - \\ &-\frac{21}{8}A^\alpha_{[\mu_1\mu_2}(V_+^\beta\ \bar\epsilon_2\gamma_{\mu_3\ldots\mu_7]}\epsilon_1 - V_-^\beta\ \bar\epsilon_{2C}\gamma_{\mu_3\ldots\mu_7]}\epsilon_{1C})\,, \end{aligned} \tag{3.44}$$

as one obtains from eqs. (3.24), (3.28) and (3.39).

4. Ten-forms

The construction of ten-forms differs in an essential way from that of the six- and eight-forms: they do not have a field strength and therefore they cannot be dual to some other form within the IIB theory. They do not have propagating degrees of freedom, since the charge associated to them must vanish. Therefore there is no a priori limit on the number of ten-forms one could introduce. Also the $\mathrm{SU}(1,1)$ representations cannot be guessed from the duality relations with lower rank forms. However, their supersymmetry transformations are well defined. We therefore proceed as before, determining the independent ten-forms from the requirement that the supersymmetry algebra must close. We want to determine the most general supersymmetry transformations for the ten-forms, compatible with $\mathrm{U}(1)$ invariance, for a given $\mathrm{SU}(1,1)$ representation. We first prove that both a doublet and a quadruplet of ten-forms are allowed, and then we discuss the claim that these are the only possible ten-forms that are compatible with all the symmetries of IIB supergravity.

4.1 The doublet of ten-forms

We want to determine the supersymmetry transformations of a doublet of ten-forms $A^\alpha_{\mu_1\ldots\mu_{10}}$ satisfying the reality condition

$$(A^1)^*_{\mu_1\ldots\mu_{10}} = A^2_{\mu_1\ldots\mu_{10}}\,. \tag{4.1}$$

As we have seen already in the previous sections, the supersymmetry transformation of any form consists of terms containing spinors, plus possibly terms containing lower-rank forms and their supersymmetry transformations. In the case of the ten-form doublet, U(1) invariance requires that the most general fermionic part in the supersymmetry transformation of the ten-form doublet is

$$\begin{aligned}\delta A^\alpha_{\mu_1\dots\mu_{10}} &= a\ V^\alpha_-\ \bar\epsilon\gamma_{\mu_1\dots\mu_{10}}\lambda + a^*\ V^\alpha_+\ \bar\epsilon_C\gamma_{\mu_1\dots\mu_{10}}\lambda_C + \\ &\quad + b\ V^\alpha_-\ \bar\epsilon_C\gamma_{[\mu_1\dots\mu_9}\psi_{\mu_{10}]} - b^*\ V^\alpha_+\ \bar\epsilon\gamma_{[\mu_1\dots\mu_9}\psi_{\mu_{10}]C}\,. \end{aligned} \tag{4.2}$$

The commutator of two such transformations contains the ten-form gauge transformation

$$\begin{aligned}\delta A^\alpha_{\mu_1\dots\mu_{10}} &= 10\partial_{[\mu_1}\Lambda^\alpha_{\mu_2\dots\mu_{10}]} \\ &= -20i\partial_{[\mu_1}\big(a\ V^\alpha_+\ \bar\epsilon_2\gamma_{\mu_2\dots\gamma_{10}]}\epsilon_{1C} + a^*\ V^\alpha_-\ \bar\epsilon_{2C}\gamma_{\mu_2\dots\gamma_{10}]}\epsilon_1\big)\,, \end{aligned} \tag{4.3}$$

provided that the coefficients a and b satisfy

$$b = 20ia^*\,. \tag{4.4}$$

Moreover, the additional terms in the commutator, containing the five-form F_5 and the complex three-form G_3, vanish if a is chosen to be real.

In order to close the algebra, one also has to produce a general coordinate transformation with parameter ξ^μ (2.26), but this exactly cancels with the gauge transformation of parameter[6]

$$\Lambda'^\alpha_{\mu_1\dots\mu_9} = A^\alpha_{\mu_1\dots\mu_9\sigma}\xi^\sigma\,. \tag{4.5}$$

As a result, the algebra closes without adding any term containing lower-rank forms in the supersymmetry transformation of eq. (4.2). Correspondingly, this ten-form doublet is invariant with respect to the gauge transformations of the lower-rank forms. Without loss of generality, we can fix

$$a = 1\,, \tag{4.6}$$

so that the resulting supersymmetry transformation for the ten-form doublet is

$$\begin{aligned}\delta A^\alpha_{\mu_1\dots\mu_{10}} &= V^\alpha_-\ \bar\epsilon\gamma_{\mu_1\dots\mu_{10}}\lambda + V^\alpha_+\ \bar\epsilon_C\gamma_{\mu_1\dots\mu_{10}}\lambda_C + \\ &\quad + 20i\ V^\alpha_-\ \bar\epsilon_C\gamma_{[\mu_1\dots\mu_9}\psi_{\mu_{10}]} + 20i\ V^\alpha_+\ \bar\epsilon\gamma_{[\mu_1\dots\mu_9}\psi_{\mu_{10}]C}\,. \end{aligned} \tag{4.7}$$

4.2 The quadruplet of ten-forms

We consider now a quadruplet of ten-forms $A^{\alpha\beta\gamma}_{\mu_1\dots\mu_{10}}$, completely symmetric in α, β and γ, and satisfying the reality condition

$$(A^{111})^*_{\mu_1\dots\mu_{10}} = A^{222}_{\mu_1\dots\mu_{10}}\,, \qquad (A^{112})^*_{\mu_1\dots\mu_{10}} = A^{122}_{\mu_1\dots\mu_{10}}\,. \tag{4.8}$$

[6]For lower rank p-forms these transformations are obtained in the form $\xi^\rho F_{\rho\mu_1\dots\mu_p}$, for $p = D$ the vanishing of the $D+1$-form F corresponds to the cancellation of the two transformations. This result will be used again in the next subsection, when we will consider ten-forms in other representations of $SU(1,1)$.

The most general supersymmetry transformation, compatible with the reality condition and with U(1) invariance, and consisting of terms that only involve the spinors and terms containing the lower rank forms and their supersymmetry transformations, is

$$\begin{aligned}\delta A^{\alpha\beta\gamma}_{\mu_1\ldots\mu_{10}} &= a\ V_+^{(\alpha}V_+^{\beta}V_-^{\gamma)}\ \bar\epsilon_C\gamma_{\mu_1\ldots\mu_{10}}\lambda_C + a^*\ V_-^{(\alpha}V_-^{\beta}V_+^{\gamma)}\ \bar\epsilon\gamma_{\mu_1\ldots\mu_{10}}\lambda + \\ &+b\ V_+^{(\alpha}V_+^{\beta}V_-^{\gamma)}\ \bar\epsilon\gamma_{[\mu_1\ldots\mu_9}\psi_{\mu_{10}]C} - b^*\ V_-^{(\alpha}V_-^{\beta}V_+^{\gamma)}\ \bar\epsilon_C\gamma_{[\mu_1\ldots\mu_9}\psi_{\mu_{10}]} + \\ &+c\ A^{(\alpha\beta}_{[\mu_1\ldots\mu_8}\delta A^{\gamma)}_{\mu_9\mu_{10}]} + d\ A^{(\alpha}_{[\mu_1\mu_2}\delta A^{\beta\gamma)}_{\mu_3\ldots\mu_{10}]} + e\ A^{(\alpha}_{[\mu_1\ldots\mu_6}A^{\beta}_{\mu_7\mu_8}\delta A^{\gamma)}_{\mu_9\mu_{10}]} + \\ &+f\ A^{(\alpha}_{[\mu_1\mu_2}A^{\beta}_{\mu_3\mu_4}\delta A^{\gamma)}_{\mu_5\ldots\mu_{10}]} + g\ A_{[\mu_1\ldots\mu_4}A^{(\alpha}_{\mu_5\mu_6}A^{\beta}_{\mu_7\mu_8}\delta A^{\gamma)}_{\mu_9\mu_{10}]} + \\ &+h\ A^{(\alpha}_{[\mu_1\mu_2}A^{\beta}_{\mu_3\mu_4}A^{\gamma)}_{\mu_5\mu_6}\delta A_{\mu_7\ldots\mu_{10}]} + ik\ A^{(\alpha}_{[\mu_1\mu_2}A^{\beta}_{\mu_3\mu_4}A^{\gamma)}_{\mu_5\mu_6}\epsilon_{\delta\tau}A^{\delta}_{\mu_7\mu_8}\delta A^{\tau}_{\mu_9\mu_{10}]}\,.\end{aligned} \tag{4.9}$$

We want to analyze the commutator of two such transformations.

We first consider the contribution coming from the fermionic terms, i.e., the first two lines of eq. (4.9). Those produce the ten-form gauge transformation

$$\begin{aligned}\delta A^{\alpha\beta\gamma}_{\mu_1\ldots\mu_{10}} &= 10\partial_{[\mu_1}\Lambda^{\alpha\beta\gamma}_{\mu_2\ldots\mu_{10}]} \\ &= \frac{20}{3}i\partial_{[\mu_1}(a\ V_+^{(\alpha}V_+^{\beta}V_-^{\gamma)}\ \bar\epsilon_2\gamma_{\mu_2\ldots\mu_{10}]}\epsilon_{1C} + a^*\ V_-^{(\alpha}V_-^{\beta}V_+^{\gamma)}\ \bar\epsilon_{2C}\gamma_{\mu_2\ldots\mu_{10}]}\epsilon_1)\end{aligned} \tag{4.10}$$

together with the terms

$$\begin{aligned}&\frac{20}{3}a\ F^{(\alpha\beta}_{[\mu_1\ldots\mu_9}\left(V_+^{\gamma)}\ \bar\epsilon_2\gamma_{\mu_{10}]}\epsilon_{1C} + V_-^{\gamma)}\ \bar\epsilon_{2C}\gamma_{\mu_{10}]}\epsilon_1\right) - \\ &-20ai\ S^{(\alpha\beta}\left(\bar\epsilon_2\gamma_{[\mu_1\ldots\mu_7}\epsilon_1 - \bar\epsilon_{2C}\gamma_{[\mu_1\ldots\mu_7}\epsilon_{1C}\right)F^{\gamma)}_{\mu_8\mu_9\mu_{10}]}\,,\end{aligned} \tag{4.11}$$

provided that

$$-\frac{20i}{3}a = b \tag{4.12}$$

and a is chosen to be imaginary. Without loss of generality, we can fix

$$a = i \tag{4.13}$$

from now on. As in the case of the ten-form doublet of the previous subsection, a general coordinate transformation is automatically produced by means of a compensating gauge transformation of parameter

$$\Lambda'^{\alpha\beta\gamma}_{\mu_1\ldots\mu_9} = A^{\alpha\beta\gamma}_{\mu_1\ldots\mu_9\sigma}\xi^\sigma\,. \tag{4.14}$$

We assume that the ten-form quadruplet transforms non-trivially with respect to the lower-rank form gauge transformations, and in particular we make the Ansatz

$$\delta A^{\alpha\beta\gamma}_{\mu_1\ldots\mu_{10}} = \alpha F^{(\alpha\beta}_{[\mu_1\ldots\mu_9}\Lambda^{\gamma)}_{\mu_{10}]} + \beta F^{(\alpha}_{[\mu_1\mu_2\mu_3}\Lambda^{\beta\gamma)}_{\mu_4\ldots\mu_{10}]}\,. \tag{4.15}$$

We will comment on this choice at the end of this subsection. We now proceed exactly as in the previous cases, considering the terms in the commutator coming from the last three lines of eq. (4.9), as well as the two terms in eq. (4.11). Those have to generate the gauge transformations of eq. (4.15), possibly together with an additional ten-form gauge transformation. The final result is that the ten-form gauge transformation of parameter

$$\begin{aligned}\Lambda''^{\alpha\beta\gamma}_{\mu_1\dots\mu_9} = & -\frac{2i}{5}c\ A^{(\alpha\beta}_{[\mu_1\dots\mu_8}\left(V^{\gamma)}_+\ \bar{\epsilon}_2\gamma_{\mu_9]}\epsilon_{1C} + V^{\gamma)}_-\ \bar{\epsilon}_{2C}\gamma_{\mu_9]}\epsilon_1\right) - \\ & -\frac{2}{5}d\ A^{(\alpha}_{[\mu_1\mu_2}S^{\beta\gamma)}\left(\bar{\epsilon}_2\gamma_{\mu_3\dots\mu_9]}\epsilon_1 - \bar{\epsilon}_{2C}\gamma_{\mu_3\dots\mu_9]}\epsilon_{1C}\right)\end{aligned} \tag{4.16}$$

is produced, while the coefficients are determined to be

$$\begin{aligned}&\alpha = -\frac{2}{3}, \qquad \beta = 32, \qquad c = -12, \\ &d = 3, \qquad e = -\frac{63}{4}, \qquad f = \frac{21}{4}, \\ &g = -210, \qquad h = 105, \qquad k = \frac{315}{8}.\end{aligned} \tag{4.17}$$

Summarizing, the supersymmetry transformation of the ten-form quadruplet is

$$\begin{aligned}\delta A^{\alpha\beta\gamma}_{\mu_1\dots\mu_{10}} = & \ i\ V^{(\alpha}_+V^{\beta}_+V^{\gamma)}_-\ \bar{\epsilon}_C\gamma_{\mu_1\dots\mu_{10}}\lambda_C - i\ V^{(\alpha}_-V^{\beta}_-V^{\gamma)}_+\ \bar{\epsilon}\gamma_{\mu_1\dots\mu_{10}}\lambda + \\ & +\frac{20}{3}\ V^{(\alpha}_+V^{\beta}_+V^{\gamma)}_-\ \bar{\epsilon}\gamma_{[\mu_1\dots\mu_9}\psi_{\mu_{10}]C} - \frac{20}{3}\ V^{(\alpha}_-V^{\beta}_-V^{\gamma)}_+\ \bar{\epsilon}_C\gamma_{[\mu_1\dots\mu_9}\psi_{\mu_{10}]} - \\ & -12\ A^{(\alpha\beta}_{[\mu_1\dots\mu_8}\delta A^{\gamma)}_{\mu_9\mu_{10}]} + 3\ A^{(\alpha}_{[\mu_1\mu_2}\delta A^{\beta\gamma)}_{\mu_3\dots\mu_{10}]} - \frac{63}{4}\ A^{(\alpha}_{[\mu_1\dots\mu_6}A^{\beta}_{\mu_7\mu_8}\delta A^{\gamma)}_{\mu_9\mu_{10}]} + \\ & +\frac{21}{4}\ A^{(\alpha}_{[\mu_1\mu_2}A^{\beta}_{\mu_3\mu_4}\delta A^{\gamma)}_{\mu_5\dots\mu_{10}]} - 210\ A_{[\mu_1\dots\mu_4}A^{(\alpha}_{\mu_5\mu_6}A^{\beta}_{\mu_7\mu_8}\delta A^{\gamma)}_{\mu_9\mu_{10}]} + \\ & +105\ A^{(\alpha}_{[\mu_1\mu_2}A^{\beta}_{\mu_3\mu_4}A^{\gamma)}_{\mu_5\mu_6}\delta A_{\mu_7\dots\mu_{10}]} + \frac{315i}{8}\ A^{(\alpha}_{[\mu_1\mu_2}A^{\beta}_{\mu_3\mu_4}A^{\gamma)}_{\mu_5\mu_6}\epsilon_{\delta\tau}A^{\delta}_{\mu_7\mu_8}\delta A^{\tau}_{\mu_9\mu_{10}]}\,,\end{aligned} \tag{4.18}$$

while its gauge transformation is

$$\delta A^{\alpha\beta\gamma}_{\mu_1\dots\mu_{10}} = 10\partial_{[\mu_1}\Lambda^{\alpha\beta\gamma}_{\mu_2\dots\mu_{10}]} - \frac{2}{3}F^{(\alpha\beta}_{[\mu_1\dots\mu_9}\Lambda^{\gamma)}_{\mu_{10}]} + 32F^{(\alpha}_{[\mu_1\mu_2\mu_3}\Lambda^{\beta\gamma)}_{\mu_4\dots\mu_{10}]}\,. \tag{4.19}$$

Finally, the ten-form gauge transformation parameter appearing in the supersymmetry algebra is

$$\begin{aligned}\Lambda^{\alpha\beta\gamma}_{\mu_1\dots\mu_9} = & \ A^{\alpha\beta\gamma}_{\mu_1\dots\mu_9\sigma}\xi^{\sigma} - \frac{2}{3}(V^{(\alpha}_+V^{\beta}_+V^{\gamma)}_-\ \bar{\epsilon}_2\gamma_{\mu_1\dots\mu_9}\epsilon_{1C} - V^{(\alpha}_-V^{\beta}_-V^{\gamma)}_+\ \bar{\epsilon}_{2C}\gamma_{\mu_1\dots\mu_9}\epsilon_1) + \\ & +\frac{24i}{5}\ A^{(\alpha\beta}_{[\mu_1\dots\mu_8}\left(V^{\gamma)}_+\ \bar{\epsilon}_2\gamma_{\mu_9]}\epsilon_{1C} + V^{\gamma)}_-\ \bar{\epsilon}_{2C}\gamma_{\mu_9]}\epsilon_1\right) - \\ & -\frac{6}{5}\ A^{(\alpha}_{[\mu_1\mu_2}S^{\beta\gamma)}\left(\bar{\epsilon}_2\gamma_{\mu_3\dots\mu_9]}\epsilon_1 - \bar{\epsilon}_{2C}\gamma_{\mu_3\dots\mu_9]}\epsilon_{1C}\right)\,,\end{aligned} \tag{4.20}$$

as it results from eqs. (4.10), (4.14) and (4.16).

To conclude this subsection, we want to comment on the bosonic gauge transformation of eq. (4.19). Even though the supersymmetry algebra restricts us in our case to ten dimensions, it turns out that the bosonic gauge algebra closes for *arbitrary* dimension. In particular one can write down an eleven-form field strength that is gauge invariant with respect to a gauge transformation of the form (4.15):

$$F^{\alpha\beta\gamma}_{\mu_1\dots\mu_{11}} = 11\partial_{[\mu_1}A^{\alpha\beta\gamma}_{\mu_2\dots\mu_{11}]} + \frac{11}{2}\alpha\ A^{(\alpha}_{[\mu_1\mu_2}F^{\beta\gamma)}_{\mu_3\dots\mu_{11}]} + \frac{11}{8}\beta\ A^{(\alpha\beta}_{[\mu_1\dots\mu_8}F^{\gamma)}_{\mu_9\mu_{10}\mu_{11}]}\,, \tag{4.21}$$

where the coefficients α and β have to satisfy the constraint

$$\beta = -48\alpha\,. \tag{4.22}$$

This relation is in agreement with the values of α and β given in eq. (4.17) and obtained imposing supersymmetry. This suggests that the bosonic gauge algebra has an underlying structure that is independent of supersymmetry in ten dimensions.[7]

4.3 Other ten-forms?

We now want to show that no other ten-forms can be included in the supersymmetry algebra of IIB supergravity. In order to do this, we consider the most general Ansatz for the supersymmetry transformation of a ten-form in a generic representation of SU(1,1). Without loss of generality, we can limit ourselves to ten-forms with vanishing U(1)-charge. The simplest such example is a singlet of SU(1,1), for which the supersymmetry transformation necessarily is

$$\delta A_{\mu_1\ldots\mu_{10}} = \bar{\epsilon}\gamma_{[\mu_1\ldots\mu_9}\psi_{\mu_{10}]} + \bar{\epsilon}_C\gamma_{[\mu_1\ldots\mu_9}\psi_{\mu_{10}]C}\,. \tag{4.23}$$

The commutator of two such transformations closes. This is not surprising since $A_{(10)}$ is the volume form,

$$A_{\mu_1\ldots\mu_{10}} \propto \epsilon_{\mu_1\ldots\mu_{10}} = e_{\mu_1}{}^{a_1}\ldots e_{\mu_{10}}{}^{a_{10}}\epsilon_{a_1\ldots a_{10}}\,. \tag{4.24}$$

This means that there are no *independent* ten-form singlets in the supersymmetry algebra of IIB.

One could ask whether additional ten-form doublets could result from objects of the form $A^{\alpha_1\ldots\alpha_{2n+1}}_{\mu_1\ldots\mu_{10}}$, when $2n$ SU(1,1) indices are pairwise antisymmetrized. However, because of the constraint of eq. (2.2) these forms are the same as the one we obtained in section 4.1, and therefore there is only a single doublet of ten-forms in the theory. This argument can be iterated, so that for each object with an odd number of SU(1,1) indices, only the components in the completely symmetric representation are independent of the ten-forms belonging to lower representations.

Therefore, given a ten-form with n SU(1,1) indices, one has to consider only the completely symmetric SU(1,1) representation. Let us consider the case $n = 2$ first. The most general Ansatz for the fermionic terms is

$$\delta A^{\alpha\beta}_{(10)} = a_1 V_+^{(\alpha}V_-^{\beta)}\bar{\epsilon}\gamma_{(9)}\psi + a_2 V_+^{(\alpha}V_-^{\beta)}\bar{\epsilon}_C\gamma_{(9)}\psi_C + b_1 V_+^{\alpha}V_+^{\beta}\bar{\epsilon}\gamma_{(10)}\lambda_C + b_2 V_-^{\alpha}V_-^{\beta}\bar{\epsilon}_C\gamma_{(10)}\lambda\,. \tag{4.25}$$

As in the case of the singlet, one can close the algebra on this Ansatz, but again it is not an independent field. It turns out to be the variation of a composite field:

$$\delta\left(\frac{1}{2}S^{\alpha\beta}\epsilon_{(10)}\right) = \delta\left(V_+^{(\alpha}V_-^{\beta)}\epsilon_{(10)}\right) = V_+^{(\alpha}V_-^{\beta)}\delta\epsilon_{(10)} + \delta V_+^{(\alpha}V_-^{\beta)}\epsilon_{(10)} + V_+^{(\alpha}\delta V_-^{\beta)}\epsilon_{(10)}\,. \tag{4.26}$$

This generalises to ten-forms with $n = 2m$ SU(1,1)-indices, for which we can also close the algebra, but end up with the variation of the composite field

$$S^{(\alpha_1\beta_1}\ldots S^{\alpha_m\beta_m)}\epsilon_{(10)}\,. \tag{4.27}$$

[7] This type of gauge algebra is also observed in the doubled fields approach, see [20].

The case of n odd is different, since the requirement of vanishing U(1) charge does not allow one to write down a volume form. In this case the Ansatz for the fermionic part of the supersymmetry transformation is (we set here $n = 2m+1$)

$$\begin{aligned}
\delta A^{\alpha_1\ldots\alpha_{2m+1}}_{\mu_1\ldots\mu_{10}} = & \, a\ V_+^{(\alpha_1}\ldots V_+^{\alpha_{m+1}} V_-^{\alpha_{m+2}}\ldots V_-^{\alpha_{2m+1})}\ \bar\epsilon_C\gamma_{\mu_1\ldots\mu_{10}}\lambda_C + \\
& + a^*\ V_-^{(\alpha_1}\ldots V_-^{\alpha_{m+1}} V_+^{\alpha_{m+2}}\ldots V_+^{\alpha_{2m+1})}\ \bar\epsilon\gamma_{\mu_1\ldots\mu_{10}}\lambda + \\
& + b\ V_+^{(\alpha_1}\ldots V_+^{\alpha_{m+1}} V_-^{\alpha_{m+2}}\ldots V_-^{\alpha_{2m+1})}\ \bar\epsilon\gamma_{[\mu_1\ldots\mu_9}\psi_{\mu_{10}]C} - \\
& - b^*\ V_-^{(\alpha_1}\ldots V_-^{\alpha_{m+1}} V_+^{\alpha_{m+2}}\ldots V_+^{\alpha_{2m+1})}\ \bar\epsilon_C\gamma_{[\mu_1\ldots\mu_9}\psi_{\mu_{10}]}\,. \qquad (4.28)
\end{aligned}$$

It can be shown that only for the case $m = 0$, i.e., the doublet that we already considered, the commutator of two such transformations closes producing just a ten-form gauge transformation and a general coordinate transformation. As we have seen already for the quadruplet ($m = 1$), extra terms are generated that need to combine with additional terms in eq. (4.28), containing lower-rank forms and their supersymmetry transformations, to produce bosonic gauge transformations. An explicit analysis shows that these terms can only be written for the quadruplet. Higher SU(1,1) representations require introducing additional contributions from the scalars in these bosonic terms, and the supersymmetry commutator produces derivatives of these scalars. These contributions can not be identified with any parameter that appears in the supersymmetry algebra. This suggests that only a doublet and a quadruplet can be consistently included in the supersymmetry algebra of IIB.

5. The complete IIB transformation rules and algebra

This section collects our results for the SU(1,1)-democratic version of $D = 10$ IIB supergravity. We present the supersymmetry transformation rules, the transformation rules of the p-forms under bosonic gauge transformations, the definition of gauge invariant curvatures, and finally the results for the commutator of two supersymmetry transformations. Of course all the transformations and definitions are interdependent. All results have been derived only up to the quadratic order in the fermions.

The supersymmetry transformation rules in Einstein frame, in the notation of [1, 2], are:

$$\begin{aligned}
\delta e_\mu{}^a &= i\bar\epsilon\gamma^a\psi_\mu + i\bar\epsilon_C\gamma^a\psi_{\mu C}\,, && (5.1)\\
\delta\psi_\mu &= D_\mu\epsilon + \frac{i}{480}F_{\mu\nu_1\ldots\nu_4}\gamma^{\nu_1\ldots\nu_4}\epsilon + \frac{1}{96}G^{\nu\rho\sigma}\gamma_{\mu\nu\rho\sigma}\epsilon_C - \frac{3}{32}G_{\mu\nu\rho}\gamma^{\nu\rho}\epsilon_C\,, && (5.2)\\
\delta A^\alpha_{\mu\nu} &= V^\alpha_-\ \bar\epsilon\gamma_{\mu\nu}\lambda + V^\alpha_+\ \bar\epsilon_C\gamma_{\mu\nu}\lambda_C + 4iV^\alpha_-\ \bar\epsilon_C\gamma_{[\mu}\psi_{\nu]} + 4iV^\alpha_+\ \bar\epsilon\gamma_{[\mu}\psi_{\nu]C}\,, && (5.3)\\
\delta A_{\mu\nu\rho\sigma} &= \bar\epsilon\gamma_{[\mu\nu\rho}\psi_{\sigma]} - \bar\epsilon_C\gamma_{[\mu\nu\rho}\psi_{\sigma]C} - \frac{3i}{8}\epsilon_{\alpha\beta}A^\alpha_{[\mu\nu}\delta A^\beta_{\rho\sigma]}\,, && (5.4)\\
\delta\lambda &= iP_\mu\gamma^\mu\epsilon_C - \frac{i}{24}G_{\mu\nu\rho}\gamma^{\mu\nu\rho}\epsilon\,, && (5.5)\\
\delta V^\alpha_+ &= V^\alpha_-\ \bar\epsilon_C\lambda\,, && (5.6)\\
\delta V^\alpha_- &= V^\alpha_+\ \bar\epsilon\lambda_C\,, && (5.7)
\end{aligned}$$

$$
\begin{aligned}
\delta A^{\alpha}_{\mu_1\ldots\mu_6} &= iV_-^{\alpha}\bar\epsilon\gamma_{\mu_1\ldots\mu_6}\lambda - iV_+^{\alpha}\bar\epsilon_C\gamma_{\mu_1\ldots\mu_6}\lambda_C + \\
&+12\left(V_-^{\alpha}\bar\epsilon_C\gamma_{[\mu_1\ldots\mu_5}\psi_{\mu_6]} - V_+^{\alpha}\bar\epsilon\gamma_{[\mu_1\ldots\mu_5}\psi_{C\,\mu_6]}\right) + \\
&+40A_{[\mu_1\ldots\mu_4}\delta A^{\alpha}_{\mu_5\mu_6]} - 20\delta A_{[\mu_1\ldots\mu_4}A^{\alpha}_{\mu_5\mu_6]} - \\
&-\frac{15i}{2}A^{\alpha}_{[\mu_1\mu_2}\epsilon_{\beta\gamma}A^{\beta}_{\mu_3\mu_4}\delta A^{\gamma}_{\mu_5\mu_6]}\,, \qquad (5.8)\\
\delta A^{\alpha\beta}_{\mu_1\ldots\mu_8} &= +iV_-^{(\alpha}V_-^{\beta)}\bar\epsilon_C\gamma_{\mu_1\ldots\mu_8}\lambda - iV_+^{(\alpha}V_+^{\beta)}\bar\epsilon\gamma_{\mu_1\ldots\mu_8}\lambda_C + \\
&+8V_+^{(\alpha}V_-^{\beta)}(\bar\epsilon\gamma_{[\mu_1\ldots\mu_7}\psi_{\mu_8]} - \bar\epsilon_C\gamma_{[\mu_1\ldots\mu_7}\psi_{C\,\mu_8]}) + \\
&+\frac{21}{4}A^{(\alpha}_{[\mu_1\ldots\mu_6}\delta A^{\beta)}_{\mu_7\mu_8]} - \frac{7}{4}A^{(\alpha}_{[\mu_1\mu_2}\delta A^{\beta)}_{\mu_3\ldots\mu_8]} - \\
&-35A^{(\alpha}_{[\mu_1\mu_2}A^{\beta)}_{\mu_3\mu_4}\delta A_{\mu_5\ldots\mu_8]} + 70A_{[\mu_1\ldots\mu_4}A^{(\alpha}_{\mu_5\mu_6}\delta A^{\beta)}_{\mu_7\mu_8]} - \\
&-\frac{105i}{8}A^{(\alpha}_{[\mu_1\mu_2}A^{\beta)}_{\mu_3\mu_4}\epsilon_{\gamma\delta}A^{\gamma}_{\mu_5\mu_6}\delta A^{\delta}_{\mu_7\mu_8]}\,, \qquad (5.9)\\
\delta A^{\alpha}_{\mu_1\ldots\mu_{10}} &= V_-^{\alpha}\bar\epsilon\gamma_{\mu_1\ldots\mu_{10}}\lambda + V_+^{\alpha}\bar\epsilon_C\gamma_{\mu_1\ldots\mu_{10}}\lambda_C + \\
&+20i\left(V_+^{\alpha}\bar\epsilon\gamma_{[\mu_1\ldots\mu_9}\psi_{C\,\mu_{10}]} + V_-^{\alpha}\bar\epsilon_C\gamma_{[\mu_1\ldots\mu_9}\psi_{\mu_{10}]}\right), \qquad (5.10)\\
\delta A^{\alpha\beta\gamma}_{\mu_1\ldots\mu_{10}} &= iV_+^{(\alpha}V_+^{\beta}V_-^{\gamma)}\bar\epsilon_C\gamma_{\mu_1\ldots\mu_{10}}\lambda_C - iV_-^{(\alpha}V_-^{\beta}V_+^{\gamma)}\bar\epsilon\gamma_{\mu_1\ldots\mu_{10}}\lambda + \\
&+\frac{20}{3}(V_+^{(\alpha}V_+^{\beta}V_-^{\gamma)}\bar\epsilon\gamma_{[\mu_1\ldots\mu_9}\psi_{C\,\mu_{10}]} - V_-^{(\alpha}V_-^{\beta}V_+^{\gamma)}\bar\epsilon_C\gamma_{[\mu_1\ldots\mu_9}\psi_{\mu_{10}]}) - \\
&-12\,A^{(\alpha\beta}_{[\mu_1\ldots\mu_8}\delta A^{\gamma)}_{\mu_9\mu_{10}]} + 3\,A^{(\alpha}_{[\mu_1\mu_2}\delta A^{\beta\gamma)}_{\mu_3\ldots\mu_{10}]} - \\
&-\frac{63}{4}A^{(\alpha}_{[\mu_1\ldots\mu_6}A^{\beta}_{\mu_7\mu_8}\delta A^{\gamma)}_{\mu_9\mu_{10}]} + \frac{21}{4}\,A^{(\alpha}_{[\mu_1\mu_2}A^{\beta}_{\mu_3\mu_4}\delta A^{\gamma)}_{\mu_5\ldots\mu_{10}]} - \\
&-210A_{[\mu_1\ldots\mu_4}A^{(\alpha}_{\mu_5\mu_6}A^{\beta}_{\mu_7\mu_8}\delta A^{\gamma)}_{\mu_9\mu_{10}]} + 105A^{(\alpha}_{[\mu_1\mu_2}A^{\beta}_{\mu_3\mu_4}A^{\gamma)}_{\mu_5\mu_6}\delta A_{\mu_7\ldots\mu_{10}]} + \\
&+\frac{315i}{8}\,A^{(\alpha}_{[\mu_1\mu_2}A^{\beta}_{\mu_3\mu_4}A^{\gamma)}_{\mu_5\mu_6}\epsilon_{\delta\tau}A^{\delta}_{\mu_7\mu_8}\delta A^{\tau}_{\mu_9\mu_{10}]}\,. \qquad (5.11)
\end{aligned}
$$

For the bosonic gauge-transformations we find:

$$
\begin{aligned}
\delta A^{\alpha}_{\mu_1\mu_2} &= 2\partial_{[\mu_1}\Lambda^{\alpha}_{\mu_2]}\,, &(5.12)\\
\delta A_{\mu_1\ldots\mu_4} &= 4\partial_{[\mu_1}\Lambda_{\mu_2\mu_3\mu_4]} - \frac{i}{4}\epsilon_{\gamma\delta}\Lambda^{\gamma}_{[\mu_1}F^{\delta}_{\mu_2\mu_3\mu_4]}\,, &(5.13)\\
\delta A^{\alpha}_{\mu_1\ldots\mu_6} &= 6\partial_{[\mu_1}\Lambda^{\alpha}_{\mu_2\ldots\mu_6]} - 8\Lambda^{\alpha}_{[\mu_1}F_{\mu_2\ldots\mu_6]} - \frac{160}{3}F^{\alpha}_{[\mu_1\mu_2\mu_3}\Lambda_{\mu_4\mu_5\mu_6]}\,, &(5.14)\\
\delta A^{\alpha\beta}_{\mu_1\ldots\mu_8} &= 8\partial_{[\mu_1}\Lambda^{(\alpha\beta)}_{\mu_2\ldots\mu_8]} + \frac{2}{9}F^{(\alpha}_{[\mu_1\ldots\mu_7}\Lambda^{\beta)}_{\mu_8]} - \frac{14}{3}F^{(\alpha}_{[\mu_1\mu_2\mu_3}\Lambda^{\beta)}_{\mu_4\ldots\mu_8]}\,, &(5.15)\\
\delta A^{\alpha}_{\mu_1\ldots\mu_{10}} &= 10\partial_{[\mu_1}\Lambda^{\alpha}_{\mu_2\ldots\mu_{10}]}\,, &(5.16)\\
\delta A^{\alpha\beta\gamma}_{\mu_1\ldots\mu_{10}} &= 10\partial_{[\mu_1}\Lambda^{(\alpha\beta\gamma)}_{\mu_2\ldots\mu_{10}]} - \frac{2}{3}F^{(\alpha\beta}_{[\mu_1\ldots\mu_9}\Lambda^{\gamma)}_{\mu_{10}]} + 32F^{(\alpha}_{[\mu_1\mu_2\mu_3}\Lambda^{\beta\gamma)}_{\mu_4\ldots\mu_{10}]}\,. &(5.17)
\end{aligned}
$$

For the p-form fields we define field-strengths invariant under the bosonic gauge transformations

$$
\begin{aligned}
F^{\alpha}_{\mu_1\mu_2\mu_3} &= 3\partial_{[\mu_1}A^{\alpha}_{\mu_2\mu_3]}\,, &(5.18)\\
F_{\mu_1\ldots\mu_5} &= 5\partial_{[\mu_1}A_{\mu_2\ldots\mu_5]} + \frac{5i}{8}\epsilon_{\alpha\beta}A^{\alpha}_{[\mu_1\mu_2}F^{\beta}_{\mu_3\mu_4\mu_5]}\,, &(5.19)\\
F^{\alpha}_{\mu_1\ldots\mu_7} &= 7\partial_{[\mu_1}A^{\alpha}_{\mu_2\ldots\mu_7]} + 28A^{\alpha}_{[\mu_1\mu_2}F_{\mu_3\ldots\mu_7]} - \frac{280}{3}F^{\alpha}_{[\mu_1\mu_2\mu_3}A_{\mu_4\ldots\mu_7]}\,, &(5.20)
\end{aligned}
$$

$$
\begin{aligned}
F^{\alpha\beta}_{\mu_1\ldots\mu_9} &= 9\partial_{[\mu_1}A^{\alpha\beta}_{\mu_2\ldots\mu_9]} + \frac{9}{4}F^{(\alpha}_{[\mu_1\ldots\mu_7}A^{\beta)}_{\mu_8\mu_9]} - \frac{63}{4}F^{(\alpha}_{[\mu_1\mu_2\mu_3}A^{\beta)}_{\mu_4\ldots\mu_9]}\,, && (5.21)\\
F^{\alpha}_{\mu_1\ldots\mu_{11}} &= 11\partial_{[\mu_1}A^{\alpha}_{\mu_2\ldots\mu_{11}]} = 0\,, && (5.22)\\
F^{\alpha\beta\gamma}_{\mu_1\ldots\mu_{11}} &= 11(\partial_{[\mu_1}A^{\alpha\beta\gamma}_{\mu_2\ldots\mu_{11}]} - \frac{1}{3}F^{(\alpha\beta}_{[\mu_1\ldots\mu_9}A^{\gamma)}_{\mu_{10}\mu_{11}]} + 4F^{(\alpha}_{[\mu_1\mu_2\mu_3}A^{\beta\gamma)}_{\mu_4\ldots\mu_{11}[}) = 0\,. && (5.23)
\end{aligned}
$$

The duality relations between these field-strengths are:

$$
\begin{aligned}
F^{\alpha}_{\mu_1\ldots\mu_7} &= -\frac{i}{3!}\epsilon_{\mu_1\ldots\mu_7\mu\nu\rho}S^{\alpha\beta}\epsilon_{\beta\gamma}F^{\gamma;\mu\nu\rho}\,, && (5.24)\\
F^{\alpha\beta}_{\mu_1\ldots\mu_9} &= i\epsilon_{\mu_1\ldots\mu_9}{}^{\rho}[V^{\alpha}_{+}V^{\beta}_{+}P^{*}_{\rho} - V^{\alpha}_{-}V^{\beta}_{-}P_{\rho}]\,. && (5.25)
\end{aligned}
$$

The commutator of two supersymmetry transformations, $[\delta(\epsilon_1), \delta(\epsilon_2)]$ must close on symmetry transformations of the IIB multiplet. In fact, as we saw in previous sections, this is the way the results of this paper have been obtained. We find the following contributions to $[\delta(\epsilon_1), \delta(\epsilon_2)]$:

$$
\begin{aligned}
\xi^{\mu} &= i\bar{\epsilon}_2\gamma^{\mu}\epsilon_1 + i\bar{\epsilon}_{2C}\gamma^{\mu}\epsilon_{1C}\,, && (5.26)\\
\Lambda^{\alpha}_{\mu} &= A^{\alpha}_{\mu\nu}\xi^{\nu} - 2i[V^{\alpha}_{+}\bar{\epsilon}_2\gamma_{\mu}\epsilon_{1C} + V^{\alpha}_{-}\bar{\epsilon}_{2C}\gamma_{\mu}\epsilon_1]\,, && (5.27)\\
\Lambda_{\mu_1\mu_2\mu_3} &= A_{\mu_1\mu_2\mu_3\nu}\xi^{\nu} + \frac{1}{4}[\bar{\epsilon}_2\gamma_{\mu_1\mu_2\mu_3}\epsilon_1 - \bar{\epsilon}_{2C}\gamma_{\mu_1\mu_2\mu_3}\epsilon_{1C}]\,, && (5.28)\\
\Lambda^{\alpha}_{\mu_1\ldots\mu_5} &= A^{\alpha}_{\mu_1\ldots\mu_5\rho}\xi^{\rho} - 2V^{\alpha}_{-}\bar{\epsilon}_{2C}\gamma_{\mu_1\ldots\mu_5}\epsilon_1 + 2V^{\alpha}_{+}\bar{\epsilon}_2\gamma_{\mu_1\ldots\mu_5}\epsilon_{1C} + \\
&\quad + \frac{40}{3}A_{[\mu_1\ldots\mu_4}\Lambda^{\alpha}_{\mu_5]} - \frac{40}{3}\Lambda_{[\mu_1\mu_2\mu_3}A^{\alpha}_{\mu_4\mu_5]}\,, && (5.29)\\
\Lambda^{\alpha\beta}_{\mu_1\ldots\mu_7} &= A^{(\alpha\beta)}_{\mu_1\ldots\mu_7\nu}\xi^{\nu} - 2V^{(\alpha}_{+}V^{\beta)}_{-}\,(\bar{\epsilon}_2\gamma_{\mu_1\ldots\mu_7}\epsilon_1 - \bar{\epsilon}_{2C}\gamma_{\mu_1\ldots\mu_7}\epsilon_{1C}) + \\
&\quad + \frac{21}{16}A^{(\alpha}_{[\mu_1\ldots\mu_6}\Lambda^{\beta)}_{\mu_7]} - \frac{21}{16}A^{(\alpha}_{[\mu_1\mu_2}\Lambda^{\beta)}_{\mu_3\ldots\mu_7]}\,, && (5.30)\\
\Lambda^{\alpha}_{\mu_1\ldots\mu_9} &= -2i\left(V^{\alpha}_{+}\bar{\epsilon}_2\gamma_{\mu_1\ldots\mu_9}\epsilon_{1C} + \;V^{\alpha}_{-}\bar{\epsilon}_{2C}\gamma_{\mu_1\ldots\mu_9}\epsilon_1\right) && (5.31)\\
\Lambda^{\alpha\beta\gamma}_{\mu_1\ldots\mu_9} &= A^{(\alpha\beta\gamma)}_{\mu_1\ldots\mu_9\nu}\xi^{\nu} - \frac{2}{3}\left(V^{(\alpha}_{+}V^{\beta}_{+}V^{\gamma)}_{-}\bar{\epsilon}_2\gamma_{\mu_1\ldots\mu_9}\epsilon_{1C} - V^{(\alpha}_{-}V^{\beta}_{-}V^{\gamma)}_{+}\bar{\epsilon}_{2C}\gamma_{\mu_1\ldots\mu_9}\epsilon_1\right) + \\
&\quad + \frac{24i}{5}A^{(\alpha\beta}_{[\mu_1\ldots\mu_8}(V^{\gamma)}_{+}\bar{\epsilon}_2\gamma_{\mu_9]}\epsilon_{1C} + V^{\gamma)}_{-}\bar{\epsilon}_{2C}\gamma_{\mu_9]}\epsilon_1) - \\
&\quad - \frac{6}{5}A^{(\alpha}_{[\mu_1\mu_2}S^{\beta\gamma)}(\bar{\epsilon}_2\gamma_{\mu_3\ldots\mu_9]}\epsilon_1 - \bar{\epsilon}_{2C}\gamma_{\mu_3\ldots\mu_9]}\epsilon_{1C})\,. && (5.32)
\end{aligned}
$$

This concludes the summary of our main results. In the next section we will present the IIB supergravity multiplet in a real formulation in both Einstein frame and string frame.

6. U(1) gauge fixing and string frame

The results we derived so far were in Einstein frame. To go to string frame we will first choose a U(1) gauge, so that the dependence on the dilaton becomes explicit. Our choice is

$$
V^{1}_{-} \in \mathbb{R} \Rightarrow V^{2}_{+} = V^{1}_{-}\,. \tag{6.1}
$$

To preserve this condition the supersymmetry transformations have to be modified by a field dependent U(1) gauge transformation:

$$
\delta'(\epsilon) = \delta(\epsilon) + \delta_{\mathrm{U}(1)}\left(\frac{i}{2V^{1}_{-}}(V^{2}_{-}\bar{\epsilon}_C\lambda - V^{1}_{+}\bar{\epsilon}\lambda_C)\right)\,. \tag{6.2}
$$

This modification is only visible on the scalars since on the fermions it gives rise to terms cubic in fermionic variables. The SU(1, 1) transformations are also modified: the condition (6.1) is preserved under a combination of an SU(1, 1) and a U(1) transformation. On a field χ of U(1)-charge q the required U(1) transformation is

$$\chi \to e^{iq\theta}\chi\,, \qquad \text{with } e^{2i\theta} = \frac{\alpha + \beta z}{\bar{\alpha} + \bar{\beta} z}\,, \tag{6.3}$$

where the coordinate z is defined in (2.16). This is of course visible on all fermions.

To make the dilaton and axion explicit we set

$$V_-^1 = V_+^2 = \frac{1}{\sqrt{1 - z\bar{z}}}\,, \qquad V_-^2 = (V_+^1)^* = \frac{z}{\sqrt{1 - z\bar{z}}}\,. \tag{6.4}$$

Using (2.20) and (2.24) we find (we now drop the prime on the redefined supersymmetry transformation)

$$\delta\tau = -2ie^{-\phi}e^{-2i\Lambda}\bar{\epsilon}\lambda_C\,, \tag{6.5}$$

where

$$e^{-2i\Lambda} = \frac{1 - i\tau}{1 + i\bar{\tau}}\,. \tag{6.6}$$

Useful variables are

$$P_\mu = -\frac{i}{2}e^{-\phi}e^{-2i\Lambda}\partial_\mu\tau\,, \qquad Q_\mu = \frac{1}{4}e^{\phi}\left(\frac{1 - i\bar{\tau}}{1 - i\tau}\partial_\mu\tau + \frac{1 + i\tau}{1 + i\bar{\tau}}\partial_\mu\bar{\tau}\right)\,. \tag{6.7}$$

It is convenient to get rid of the factors of $e^{-2i\Lambda}$ in the supersymmetry transformation rules [18]. To do this we redefine the fermions by phase factors according to their U(1) weights:

$$\lambda' = e^{3i\Lambda/2}\lambda\,, \qquad \psi'_\mu = e^{i\Lambda/2}\psi_\mu\,, \qquad \epsilon' = e^{i\Lambda/2}\epsilon\,. \tag{6.8}$$

In the transformation rules the scalars $V_\pm^\alpha$ will now occur everywhere in the combination $V_\pm^\alpha e^{\pm i\Lambda}$, which are:

$$\begin{aligned}
V_-^1 e^{-i\Lambda} &= \frac{1}{2}e^{\phi/2}(1 - i\tau)\,,\\
V_+^1 e^{i\Lambda} &= \frac{1}{2}e^{\phi/2}(1 - i\bar{\tau})\,,\\
V_-^2 e^{-i\Lambda} &= \frac{1}{2}e^{\phi/2}(1 + i\tau)\,,\\
V_+^2 e^{i\Lambda} &= \frac{1}{2}e^{\phi/2}(1 + i\bar{\tau})\,.
\end{aligned} \tag{6.9}$$

Note that interchanging $V^1 \leftrightarrow V^2$ corresponds to $\tau \leftrightarrow -\tau$, $V_+ \leftrightarrow V_-$ to $\tau \leftrightarrow \bar{\tau}$. The transformation rules for the IIB supergravity multiplet of [1, 2] now become [21]:

$$\delta e_\mu{}^a = i(\bar{\epsilon}\gamma^a\psi_\mu) + \text{h.c.} \tag{6.10}$$

$$\begin{aligned}
\delta\psi_\mu &= D_\mu\epsilon - \frac{i}{4}e^{\phi}\epsilon\partial_\mu\ell + \frac{i}{480}F_{\mu_1\ldots\mu_5}\gamma^{\mu_1\ldots\mu_5}\gamma_\mu\epsilon - \\
&\quad -\frac{1}{192}e^{\phi/2}\left(\gamma_\mu\gamma^{\nu\rho\sigma} + 2\gamma^{\nu\rho\sigma}\gamma_\mu\right)\epsilon_C(F^1 - F^2 + i\bar{\tau}(F^1 + F^2))_{\nu\rho\sigma}\,,
\end{aligned} \tag{6.11}$$

$$
\begin{aligned}
\delta A^1_{\mu\nu} &= 2ie^{\phi/2}\Big\{(1-i\bar\tau)\Big(\bar\epsilon\gamma_{[\mu}\psi_{C\,\nu]} - \frac{i}{4}\bar\epsilon_C\gamma_{\mu\nu}\lambda_C\Big) + \\
&\qquad + (1-i\tau)\Big(\bar\epsilon_C\gamma_{[\mu}\psi_{\nu]} - \frac{i}{4}\epsilon\gamma_{\mu\nu]}\lambda\Big)\Big\}\,, && (6.12)\\
\delta A^2_{\mu\nu} &= 2ie^{\phi/2}\Big\{(1+i\bar\tau)\Big(\bar\epsilon\gamma_{[\mu}\psi_{C\nu]} - \frac{i}{4}\bar\epsilon_C\gamma_{\mu\nu}\lambda_C\Big) + \\
&\qquad + (1+i\tau)\Big(\bar\epsilon_C\gamma_{[\mu}\psi_{\nu]} - \frac{i}{4}\epsilon\gamma_{\mu\nu]}\lambda\Big)\Big\}\,, && (6.13)\\
\delta A_{\mu\nu\rho\sigma} &= \bar\epsilon\gamma_{[\mu\nu\rho}\psi_{\sigma]} + \text{ h.c.} - \frac{3i}{8}\epsilon_{\alpha\beta}A^\alpha_{[\mu\nu}\delta A^\beta_{\rho\sigma]}\,, && (6.14)\\
\delta\lambda &= \frac{1}{2}e^\phi\gamma^\mu\epsilon_C\partial_\mu\bar\tau - \frac{i}{48}e^{\phi/2}\gamma^{\nu\rho\sigma}\epsilon\,(F^1 - F^2 + i\bar\tau(F^1+F^2))_{\nu\rho\sigma}\,, && (6.15)\\
\delta\ell &= ie^{-\phi}(\bar\epsilon_C\lambda - \bar\epsilon\lambda_C)\,, && (6.16)\\
\delta\phi &= \bar\epsilon_C\lambda + \bar\epsilon\lambda_C\,. && (6.17)
\end{aligned}
$$

For the higher-rank form fields we present only the transformations to the fermions, because the contributions containing explicit gauge fields are unchanged by the gauge fixing and redefinitions and can be read off from (5.8)–(5.11). We find:

$$
\begin{aligned}
\delta A^1{}_{(6)} &= 6e^{\phi/2}\Big\{(1-i\tau)\Big(\bar\epsilon_C\gamma_{(5)}\psi + \frac{i}{12}\bar\epsilon\gamma_{(6)}\lambda\Big) - \\
&\qquad - (1-i\bar\tau)(\bar\epsilon\gamma_{(5)}\psi + \frac{i}{12}\bar\epsilon_C\gamma_{(6)}\lambda_C\Big)\Big\} + \ldots\,, && (6.18)\\
\delta A^2{}_{(6)} &= 6e^{\phi/2}\Big\{(1+i\tau)\Big(\bar\epsilon_C\gamma_{(5)}\psi + \frac{i}{12}\bar\epsilon\gamma_{(6)}\lambda\Big) - \\
&\qquad - (1+i\bar\tau)\Big(\bar\epsilon\gamma_{(5)}\psi_C + \frac{i}{12}\bar\epsilon_C\gamma_{(6)}\lambda_C\Big)\Big\} + \cdots\,, && (6.19)\\
\delta A^{11}{}_{(8)} &= 2e^\phi\Big\{(1-i\tau)(1-i\bar\tau)(\bar\epsilon\gamma_{(7)}\psi - \bar\epsilon_C\gamma_{(7)}\psi_C) + \\
&\qquad + \frac{i}{8}((1-i\tau)^2\bar\epsilon_C\gamma_{(8)}\lambda - (1-i\bar\tau)^2\bar\epsilon\gamma_{(8)}\lambda_C)\Big\} + \cdots\,, && (6.20)\\
\delta A^{22}{}_{(8)} &= 2e^\phi\Big\{(1+i\tau)(1+i\bar\tau)(\bar\epsilon\gamma_{(7)}\psi - \bar\epsilon_C\gamma_{(7)}\psi_C) + \\
&\qquad + \frac{i}{8}((1+i\tau)^2\bar\epsilon_C\gamma_{(8)}\lambda - (1+i\bar\tau)^2\bar\epsilon\gamma_{(8)}\lambda_C)\Big\} + \cdots\,, && (6.21)\\
\delta A^{12}{}_{(8)} &= e^\phi\Big\{((1-i\tau)(1-i\bar\tau) + (1+i\tau)(1+i\bar\tau))(\bar\epsilon\gamma_{(7)}\psi - \bar\epsilon_C\gamma_{(7)}\psi_C) + \\
&\qquad + \frac{i}{8}((1-i\tau)(1+i\tau)\bar\epsilon_C\gamma_{(8)}\lambda - (1-i\bar\tau)(1+i\bar\tau)\bar\epsilon\gamma_{(8)}\lambda_C)\Big\} + \cdots\,, && (6.22)\\
\delta A^1{}_{(10)} &= 10ie^{\phi/2}\Big\{(1-i\bar\tau)\Big(\bar\epsilon\gamma_{(9)}\psi_C - \frac{i}{20}\bar\epsilon_C\gamma_{(10)}\lambda_C\Big) + \\
&\qquad + (1-i\tau)\Big(\bar\epsilon_C\gamma_{(9)}\psi - \frac{i}{20}\bar\epsilon\gamma_{(10)}\lambda\Big)\Big\}\,, && (6.23)
\end{aligned}
$$

$$
\begin{aligned}
\delta A^2{}_{(10)} &= 10ie^{\phi/2}\left\{(1+i\bar\tau)\left(\bar\epsilon\gamma_{(9)}\psi_C - \frac{i}{20}\bar\epsilon_C\gamma_{(10)}\lambda_C\right) + \right.\\
&\qquad\qquad \left. + (1+i\tau)\left(\bar\epsilon_C\gamma_{(9)}\psi - \frac{i}{20}\bar\epsilon\gamma_{(10)}\lambda\right)\right\}, \qquad (6.24)\\
\delta A^{111}_{(10)} &= \frac{5}{6}e^{3\phi/2}(1-i\tau)(1-i\bar\tau)\{(1-i\bar\tau)(\bar\epsilon\gamma_{(9)}\psi_C + \frac{3i}{20}\bar\epsilon_C\gamma_{(10)}\lambda_C)\\
&\quad -(1-i\tau)(\bar\epsilon_C\gamma_{(9)}\psi + \frac{3i}{20}\bar\epsilon\gamma_{(10)}\lambda)\} + \cdots, \qquad (6.25)\\
\delta A^{222}_{(10)} &= \frac{5}{6}e^{3\phi/2}(1+i\tau)(1+i\bar\tau)\left\{(1+i\bar\tau)\left(\bar\epsilon\gamma_{(9)}\psi_C + \frac{3i}{20}\bar\epsilon_C\gamma_{(10)}\lambda_C\right) - \right.\\
&\qquad\qquad \left. - (1+i\tau)\left(\bar\epsilon_C\gamma_{(9)}\psi + \frac{3i}{20}\bar\epsilon\gamma_{(10)}\lambda\right)\right\} + \cdots, \qquad (6.26)\\
\delta A^{112}_{(10)} &= \frac{5}{18}e^{3\phi/2}\Big\{((1-i\bar\tau)^2(1+i\tau) + 2(1-i\tau)(1-i\bar\tau)(1+i\bar\tau)) \times\\
&\qquad \times\left(\bar\epsilon\gamma_{(10)}\psi_C + \frac{3i}{20}\bar\epsilon_C\gamma_{(10)}\lambda_C\right) -\\
&\qquad - ((1-i\tau)^2(1+i\bar\tau) + 2(1-i\tau)(1+i\tau)(1-i\bar\tau)) \times\\
&\qquad \times\left(\bar\epsilon_C\gamma_{(9)}\psi + \frac{3i}{20}\bar\epsilon\gamma_{(10)}\lambda\right)\Big\} + \cdots, \qquad (6.27)\\
\delta A^{221}_{(10)} &= \frac{5}{18}e^{3\phi/2}\Big\{((1+i\bar\tau)^2(1-i\tau) + 2(1+i\tau)(1+i\bar\tau)(1-i\bar\tau)) \times\\
&\qquad \times\left(\bar\epsilon\gamma_{(9)}\psi_C + \frac{3i}{20}\bar\epsilon_C\gamma_{(10)}\lambda_C\right)\\
&\qquad - ((1+i\tau)^2(1-i\bar\tau) + 2(1+i\tau)(1-i\tau)(1+i\bar\tau)) \times\\
&\qquad \times\left(\bar\epsilon_C\gamma_{(9)}\psi + \frac{3i}{20}\bar\epsilon\gamma_{(10)}\lambda\right)\Big\} + \cdots, \qquad (6.28)
\end{aligned}
$$

Here the dots stand for the gauge field terms given in (5.8)–(5.11). So in formulas (6.10) to (6.28) we have collected the complete set of Einstein frame supersymmetry transformations in the real formulation.

Let us now review the transformations under SU(1,1) and SL(2,$\mathbb{R}$) transformations. Consider an SU(1,1) transformation

$$
U = \begin{pmatrix} \alpha & \beta \\ \bar\beta & \bar\alpha \end{pmatrix}, \qquad \alpha\bar\alpha - \beta\bar\beta = 1. \qquad (6.29)
$$

The field τ transforms under the corresponding SL(2,$\mathbb{R}$) transformation as

$$
\begin{aligned}
\tau &\to \frac{a\tau+b}{c\tau+d}, \qquad \delta\tau \to \frac{\delta\tau}{(c\tau+d)^2}, \qquad ad-bc=1, \qquad (6.30)\\
a &= \mathrm{Re}(\alpha-\beta), \qquad d=\mathrm{Re}(\alpha+\beta), \qquad b=-\mathrm{Im}(\alpha+\beta), \qquad c=\mathrm{Im}(\alpha-\beta).
\end{aligned}
$$

The redefinition (6.8) modifies the behavior under SU(1,1) transformations. The compensating U(1) transformation on a field χ of charge q (6.3) is now changed to

$$
\chi \to e^{iq\xi}\chi, \qquad \text{with } e^{2i\xi} = \frac{c\tau+d}{c\bar\tau+d}. \qquad (6.31)
$$

For the dilaton one finds

$$e^{\phi} \to e^{\phi}\,(c\tau + d)(c\bar{\tau} + d)\,. \tag{6.32}$$

One easily verifies that, e.g., the supersymmetry variation of τ

$$\delta\tau = -2ie^{-\phi}\,\bar{\epsilon}\lambda_C \tag{6.33}$$

is consistent with these transformations. The bosonic fields with vanishing U(1) charge still transform in the standard way under SU(1,1).

We will now bring some order into the collection of higher-rank forms (6.18) to (6.28) by considering certain linear combinations of these. We choose the linear combinations of the n-forms such that for a given n, each combination has a unique power of τ in the fermionic terms of the supersymmetry variation. This is motivated by the fact that the RR-forms come with a prefactor of $e^{-\phi}$ in the standard string frame basis, which is proportional to $\tau - \bar{\tau}$. Thus we make the following definitions:

$$\begin{aligned}
\tilde{C}_{(2)} &= A^1_{(2)} - A^2_{(2)}\,, \qquad \tilde{B}_{(2)} = A^1_{(2)} + A^2_{(2)}\,, && (6.34)\\
\tilde{C}_{(4)} &= A_{(4)}\,, && (6.35)\\
\tilde{C}_{(6)} &= A^1_{(6)} + A^2_{(6)}\,, \qquad \tilde{B}_{(6)} = A^1_{(6)} - A^2_{(6)}\,, && (6.36)\\
\tilde{C}_{(8)} &= A^{11}_{(8)} + A^{22}_{(8)} + 2A^{12}_{(8)}\,, \qquad \tilde{B}_{(8)} = A^{11}_{(8)} + A^{22}_{(8)} - 2A^{12}_{(8)}\,, && (6.37)\\
\tilde{D}_{(8)} &= A^{11}_{(8)} - A^{22}_{(8)}\,, && (6.38)\\
\tilde{\mathcal{D}}_{(10)} &= A^1_{(10)} + A^2_{(10)}\,, \qquad \tilde{\mathcal{E}}_{(10)} = A^1_{(10)} - A^2_{(10)}\,, && (6.39)\\
\tilde{C}_{(10)} &= A^{111}_{(10)} + A^{222}_{(10)} + 3\left(A^{112}_{(10)} + A^{221}_{(10)}\right), && (6.40)\\
\tilde{B}_{(10)} &= A^{111}_{(10)} - A^{222}_{(10)} - 3\left(A^{112}_{(10)} - A^{221}_{(10)}\right), && (6.41)\\
\tilde{E}_{(10)} &= A^{111}_{(10)} + A^{222}_{(10)} - \left(A^{112}_{(10)} + A^{221}_{(10)}\right), && (6.42)\\
\tilde{D}_{(10)} &= A^{111}_{(10)} - A^{222}_{(10)} + \left(A^{112}_{(10)} - A^{221}_{(10)}\right). && (6.43)
\end{aligned}$$

A nice property, and partial justification why we refer to some of these linear combinations as $\tilde{C}_{(n)}$ (RR fields) and $\tilde{B}_{(n)}$ (NS-NS fields) is the way these fields transform into each other under S-duality. The discrete S-duality transformation $\tau \to -1/\tau$ corresponds to an SL$(2,\mathbb{R})$-transformation with $a = d = 0$, $b = -c = 1$. The behaviour of the form-fields under S-duality is

$$\begin{aligned}
&\tilde{C}_{(2)} \to -i\,\tilde{B}_{(2)}\,, \qquad \tilde{B}_{(2)} \to -i\,\tilde{C}_{(2)}\,,\\
&\tilde{C}_{(4)} \to \tilde{C}_{(4)}\,,\\
&\tilde{C}_{(6)} \to -i\,\tilde{B}_{(6)}\,, \qquad \tilde{B}_{(6)} \to -i\,\tilde{C}_{(6)}\,,\\
&\tilde{C}_{(8)} \to -\,\tilde{B}_{(8)}\,, \qquad \tilde{B}_{(8)} \to -\,\tilde{C}_{(8)}\,, \qquad \tilde{D}_{(8)} \to -\,\tilde{D}_{(8)}\,,\\
&\tilde{\mathcal{D}}_{(10)} \to -i\,\tilde{\mathcal{E}}_{(10)}\,, \qquad \tilde{\mathcal{E}}_{(10)} \to -i\,\tilde{\mathcal{D}}_{(10)}\,,\\
&\tilde{C}_{(10)} \to i\,\tilde{B}_{(10)}\,, \qquad \tilde{B}_{(10)} \to i\,\tilde{C}_{(10)}\,,\\
&\tilde{D}_{(10)} \to i\tilde{E}_{(10)}\,, \qquad \tilde{E}_{(10)} \to i\,\tilde{D}_{(10)}\,.
\end{aligned} \tag{6.44}$$

We see that applying S-duality twice gives $+1$ on τ and on the four- and eight-forms, but -1 on the two-, six- and ten-forms. That this is indeed right, and that the S-duality transformation is not its own inverse can be seen easily from translating back to the SU(1,1) notation via (6.30), in which the S-duality transformation matrix is given by

$$U = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix} \tag{6.45}$$

so that U^2 gives a minus on forms with an odd number of SU(1,1)-indices.

Now we are ready to transform to string frame. The basic transformation is $e_{(E)\,\mu}{}^a = e^{-\phi/4} e_{(S)\,\mu}{}^a$. We choose to write the variation of the zehnbein in standard form, which requires a modification of supersymmetry with a λ-dependent local Lorentz transformation (which we see only on the zehnbein), and a redefinition:

$$\begin{aligned} \epsilon' &= e^{\phi/8}\,\epsilon\,, && (6.46)\\ \lambda' &= e^{-\phi/8}\,\lambda\,, && (6.47)\\ \psi'_\mu &= e^{\phi/8}\psi_\mu - \frac{i}{4}\gamma'_\mu\lambda'_C\,, && (6.48)\\ \gamma'_\mu &= e^{\phi/4}\gamma_\mu\,. && (6.49) \end{aligned}$$

Again we start with the basic supergravity multiplet and then discuss the high-rank forms. The transformation rules are simplified by writing the complex fermions as real doublets, i.e. $\epsilon \to (\epsilon_1,\ \epsilon_2)$, where ϵ_i are real Majorana-Weyl fermions. This gives rise to the appearance of Pauli matrices $\sigma_0 = 1, \sigma_1, i\sigma_2, \sigma_3$ in the contractions between such doublets, generically:

$$\begin{aligned} &\bar\epsilon_C\gamma\chi + \bar\epsilon\gamma\chi_C \to 2\bar\epsilon\sigma_3\gamma\chi\,, \qquad \bar\epsilon_C\gamma\chi_C + \bar\epsilon\gamma\chi \to 2\bar\epsilon\gamma\chi\,, && (6.50)\\ &\bar\epsilon_C\gamma\chi - \bar\epsilon\gamma\chi_C \to 2i\bar\epsilon\sigma_1\gamma\chi\,, \qquad \bar\epsilon_C\gamma\chi_C - \bar\epsilon\gamma\chi \to -2i\bar\epsilon(i\sigma_2)\gamma\chi\,. && (6.51) \end{aligned}$$

In addition we redefine $\lambda \to \lambda_C$, or, equivalently, in the real notation

$$\lambda \to \sigma_3\lambda\,. \tag{6.52}$$

We drop all primes in the string frame transformation rules:

$$\begin{aligned} \delta e_\mu{}^a &= 2i\bar\epsilon\gamma^a\psi_\mu && (6.53)\\ \delta\psi_\mu &= D_\mu\epsilon + \frac{1}{8}e^\phi\gamma^\nu\partial_\nu\ell\gamma_\mu\,(i\sigma_2)\,\epsilon - \frac{1}{16}\gamma^{\nu\rho}\sigma_3\epsilon F_{+\mu\nu\rho} + \\ &\quad + \frac{i}{96}e^\phi\gamma^{\nu\rho\sigma}\gamma_\mu\,\sigma_1\,\epsilon(F_- + i\ell F_+)_{\nu\rho\sigma} - \\ &\quad - \frac{1}{480}e^\phi\gamma^{\mu_1\ldots\mu_5}\gamma_\mu\,(i\sigma_2)\,\epsilon F_{\mu_1\ldots\mu_5}\,, && (6.54)\\ \delta\tilde B_{\mu\nu} &= 8i\,\bar\epsilon\,\sigma_3\,\gamma_{[\mu}\psi_{\nu]}\,, && (6.55)\\ \delta\tilde C_{\mu\nu} &= -8e^{-\phi}\,\bar\epsilon\,\sigma_1\,\gamma_{[\mu}(\psi_{\nu]} + \frac{i}{2}\gamma_{\nu]}\lambda) - i\ell\,\delta\tilde B_{\mu\nu}\,, && (6.56)\\ \delta\tilde C_{\mu\nu\rho\sigma} &= 2ie^{-\phi}\,\bar\epsilon\,(i\sigma_2)\,\gamma_{[\mu\nu\rho}(\psi_{\sigma]} + \frac{i}{4}\gamma_{\sigma]}\lambda) - \frac{3i}{16}\{\tilde C\delta\tilde B - \tilde B\delta\tilde C\}_{\mu\nu\rho\sigma}\,, && (6.57) \end{aligned}$$

$$\begin{aligned}
\delta\lambda &= \frac{i}{2}\gamma^\mu\partial_\mu\phi\,\epsilon - \frac{i}{48}\gamma^{\nu\rho\sigma}\,\sigma_3\,\epsilon F_{+\nu\rho\sigma} - \frac{i}{2}e^\phi\gamma^\mu\partial_\mu\ell\,(i\sigma_2)\,\epsilon \\
&\quad + \frac{1}{48}e^\phi\gamma^{\nu\rho\sigma}\,\sigma_1\,\epsilon(F_- + i\ell F_+)_{\nu\rho\sigma}\,, && (6.58)\\
\delta\ell &= 2\,e^{-\phi}\bar\epsilon\,(i\sigma_2)\,\lambda\,, && (6.59)\\
\delta\phi &= 2\,\bar\epsilon\lambda && (6.60)
\end{aligned}$$

where we have defined

$$F_+ = F^1 + F^2\,, \qquad F_- = F^1 - F^2\,. \tag{6.61}$$

For the higher form fields we find:

$$\begin{aligned}
\delta\tilde C_{(6)} &= 24i\,e^{-\phi}\,\bar\epsilon\,\sigma_1\gamma_{(5)}\left(\psi + \frac{i}{6}\gamma_{(1)}\lambda\right) + 40\tilde C_{(4)}\delta\tilde B_{(2)} - \\
&\quad -20\delta\tilde C_{(4)}\tilde B_{(2)} - \frac{15i}{4}\tilde B_{(2)}\left(\tilde C_{(2)}\delta\tilde B_{(2)} - \tilde B_{(2)}\delta\tilde C_{(2)}\right), && (6.62)\\
\delta\tilde B_{(6)} &= 24\,e^{-\phi}\left\{\ell\,\bar\epsilon\,\sigma_1\gamma_{(5)}\left(\psi + \frac{i}{6}\gamma_{(1)}\lambda\right) + e^{-\phi}\,\bar\epsilon\,\sigma_3\gamma_{(5)}\left(\psi + \frac{i}{3}\gamma_{(1)}\lambda\right)\right\} + \\
&\quad +40\tilde C_{(4)}\delta\tilde C_{(2)} - 20\delta\tilde C_{(4)}\tilde C_{(2)} - \frac{15i}{4}\tilde C_{(2)}\left(\tilde C_{(2)}\delta\tilde B_{(2)} - \tilde B_{(2)}\delta\tilde C_{(2)}\right), && (6.63)\\
\delta\tilde C_{(8)} &= 16i\,e^{-\phi}\,\bar\epsilon\,(i\sigma_2)\gamma_{(7)}\left(\psi + \frac{i}{8}\gamma_{(1)}\lambda\right) + \frac{21}{4}\tilde C_{(6)}\delta\tilde B_{(2)} - \frac{7}{4}\tilde B_{(2)}\delta\tilde C_{(6)} - \\
&\quad -35\tilde B_{(2)}\tilde B_{(2)}\delta\tilde C_{(4)} + 70\tilde C_{(4)}\tilde B_{(2)}\delta\tilde B_{(2)]} - \\
&\quad -\frac{105i}{16}\tilde B_{(2)}\tilde B_{(2)}\left(\tilde C_{(2)}\delta\tilde B_{(2)} - \tilde B_{(2)}\delta\tilde C_{(2)}\right), && (6.64)\\
\delta\tilde B_{(8)} &= -16i\,e^{-\phi}\left\{\ell^2\,\bar\epsilon(i\sigma_2)\gamma_{(7)}\left(\psi + \frac{i}{8}\gamma_{(1)}\lambda\right) + \right.\\
&\qquad\qquad \left. + \frac{i}{4}\ell e^{-\phi}\,\bar\epsilon\gamma_{(8)}\lambda + e^{-2\phi}\,\bar\epsilon\,(i\sigma_2)\gamma_{(7)}\left(\psi + \frac{3i}{8}\gamma_{(1)}\lambda\right)\right\} + \\
&\quad +\frac{21}{4}\tilde B_{(6)}\delta\tilde C_{(2)} - \frac{7}{4}\tilde C_{(2)}\delta\tilde B_{(6)} - 35\tilde C_{(2)}\tilde C_{(2)}\delta\tilde C_{(4)} + \\
&\quad +70\tilde C_{(4)}\tilde C_{(2)}\delta\tilde C_{(2)} - \frac{105i}{16}\tilde C_{(2)}\tilde C_{(2)}\left(\tilde C_{(2)}\delta\tilde B_{(2)} - \tilde B_{(2)}\delta\tilde C_{(2)}\right), && (6.65)\\
\delta\tilde D_{(8)} &= 16\ell e^{-\phi}\,\bar\epsilon\,(i\sigma_2)\gamma_{(7)}\left(\psi + \frac{i}{8}\gamma_{(1)}\lambda\right) + 2ie^{-2\phi}\bar\epsilon\gamma_{(8)}\lambda + \\
&\quad +\frac{21}{8}\{\tilde C_{(6)}\delta\tilde C_{(2)} + \tilde B_{(6)}\delta\tilde B_{(2)}\} - \frac{7}{8}\{\tilde B_{(2)}\delta\tilde B_{(6)} + \tilde C_{(2)}\delta\tilde C_{(6)}\} - \\
&\quad -35\tilde B_{(2)}\tilde C_{(2)}\delta\tilde C_{(4)} + 35\tilde C_{(4)}\{\tilde B_{(2)}\delta\tilde C_{(2)} + \tilde C_{(2)}\delta\tilde B_{(2)}\} - \\
&\quad -\frac{105i}{16}\tilde C_{(2)}\tilde C_{(2)}\left(\tilde C_{(2)}\delta\tilde B_{(2)} - \tilde B_{(2)}\delta\tilde C_{(2)}\right), && (6.66)\\
\delta\tilde{\mathcal D}_{(10)} &= 40i\,e^{-2\phi}\,\bar\epsilon\,\sigma_3\gamma_{(9)}(\psi + \frac{i}{5}\gamma_{(1)}\lambda)\,, && (6.67)\\
\delta\tilde{\mathcal E}_{(10)} &= 40\,e^{-2\phi}\left\{\ell\,\bar\epsilon\,\sigma_3\gamma_{(9)}\left(\psi + \frac{i}{5}\gamma_{(1)}\lambda\right) - e^{-\phi}\,\bar\epsilon\,\sigma_1\gamma_{(9)}\left(\psi + \frac{3i}{10}\gamma_{(1)}\lambda\right)\right\}, && (6.68)\\
\delta\tilde C_{(10)} &= -\frac{40}{3}ie^{-\phi}\bar\epsilon\sigma_1\gamma_{(9)}\left(\psi + \frac{i}{10}\gamma_{(1)}\lambda\right) - && (6.69)
\end{aligned}$$

$$
\begin{aligned}
& -12\tilde{C}_{(8)}\delta\tilde{B}_{(2)} + 3\delta\tilde{C}_{(8)}\tilde{B}_{(2)} - \frac{63}{4}\tilde{C}_{(6)}\tilde{B}_{(2)}\delta\tilde{B}_{(2)} + \frac{21}{4}\delta\tilde{C}_{(6)}\tilde{B}_{(2)}\tilde{B}_{(2)} + \\
& -210\tilde{C}_{(4)}\tilde{B}_{(2)}\tilde{B}_{(2)}\delta\tilde{B}_{(2)} + 105\delta\tilde{C}_{(4)}\tilde{B}_{(2)}\tilde{B}_{(2)}\tilde{B}_{(2)} + \\
& +\frac{315}{8}\frac{1}{2}i\tilde{B}_{(2)}\tilde{B}_{(2)}\tilde{B}_{(2)}\left(\tilde{C}_{(2)}\delta\tilde{B}_{(2)} - \tilde{B}_{(2)}\delta\tilde{C}_{(2)}\right), \qquad (6.70)
\end{aligned}
$$

$$
\begin{aligned}
\delta\tilde{B}_{(10)} = & +\frac{40}{3}e^{-4\phi}\left(1+e^{2\phi}\ell^2\right)\bar{\epsilon}\sigma_3\gamma_{(9)}\left(\psi+\frac{2i}{5}\gamma_{(1)}\lambda\right) + \\
& +\frac{40}{3}\ell e^{-\phi}\left(\ell^2+e^{-2\phi}\right)\bar{\epsilon}\sigma_1\gamma_{(9)}\left(\psi+\frac{i}{10}\gamma_{(1)}\lambda\right) - \\
& -12\tilde{B}_{(8)}\delta\tilde{C}_{(2)} + 3\delta\tilde{B}_{(8)}\tilde{C}_{(2)} - \frac{63}{4}\tilde{B}_{(6)}\tilde{C}_{(2)}\delta\tilde{C}_{(2)} + \frac{21}{4}\delta\tilde{B}_{(6)}\tilde{C}_{(2)}\tilde{C}_{(2)} - \\
& -210\tilde{C}_{(4)}\tilde{C}_{(2)}\tilde{C}_{(2)}\delta\tilde{C}_{(2)} + 105\delta\tilde{C}_{(4)}\tilde{C}_{(2)}\tilde{C}_{(2)}\tilde{C}_{(2)} + \\
& +\frac{315}{8}\frac{1}{2}i\tilde{C}_{(2)}\tilde{C}_{(2)}\tilde{C}_{(2)}\left(\tilde{C}_{(2)}\delta\tilde{B}_{(2)} - \tilde{B}_{(2)}\delta\tilde{C}_{(2)}\right), \qquad (6.71)
\end{aligned}
$$

$$
\begin{aligned}
\delta\tilde{D}_{(10)} = & -\frac{40}{3}ie^{-2\phi}\bar{\epsilon}\sigma_3\gamma_{(9)}\left(\psi+\frac{2i}{5}\gamma_{(1)}\lambda\right) - \frac{40}{9}\ell e^{-\phi}\bar{\epsilon}\sigma_3\gamma_{(9)}\left(\psi+\frac{i}{10}\gamma_{(1)}\lambda\right) - \\
& -12\left(\frac{2}{3}\tilde{D}_{(8)}\delta\tilde{B}_{(2)} + \frac{1}{3}\tilde{C}_{(8)}\delta\tilde{C}_{(2)}\right) + 3\left(\frac{2}{3}\delta\tilde{D}_{(8)}\tilde{B}_{(2)} + \frac{1}{3}\delta\tilde{C}_{(8)}\tilde{C}_{(2)}\right) - \\
& -\frac{63}{4}\left(\frac{1}{3}\tilde{B}_{(6)}\tilde{C}_{(2)}\delta\tilde{B}_{(2)} + \frac{1}{3}(\tilde{B}-\tilde{C})_{(6)}(\tilde{C}-\tilde{B})_{(2)}\delta\tilde{C}_{(2)}\right) + \\
& +\frac{21}{4}\left(\frac{1}{3}\delta\tilde{B}_{(6)}\tilde{C}_{(2)}\tilde{B}_{(2)} + \frac{1}{3}\delta(\tilde{B}-\tilde{C})_{(6)}(\tilde{C}-\tilde{B})_{(2)}\tilde{C}_{(2)}\right) - \\
& -210\tilde{C}_{(4)}\left(\frac{1}{3}\delta\tilde{B}_{(2)}\tilde{C}_{(2)}\tilde{C}_{(2)} + \frac{2}{3}\delta\tilde{C}_{(2)}\tilde{B}_{(2)}\tilde{C}_{(2)}\right) + \\
& +105\delta\tilde{C}_{(4)}\left(\frac{1}{3}\tilde{B}_{(2)}\tilde{C}_{(2)}\tilde{C}_{(2)} + \frac{2}{3}\tilde{C}_{(2)}\tilde{B}_{(2)}\tilde{C}_{(2)}\right) + \\
& +\frac{315}{16}i\left(\frac{1}{3}\delta\tilde{B}_{(2)}\tilde{C}_{(2)}\tilde{C}_{(2)} + \frac{2}{3}\delta\tilde{C}_{(2)}\tilde{B}_{(2)}\tilde{C}_{(2)}\right)\left(\tilde{C}_{(2)}\delta\tilde{B}_{(2)} - \tilde{B}_{(2)}\delta\tilde{C}_{(2)}\right) \qquad (6.72)
\end{aligned}
$$

$$
\begin{aligned}
\delta\tilde{E}_{(10)} = & +\frac{40}{3}ie^{-3\phi}\left(\frac{1}{3}+\ell^2e^{2\phi}\right)\bar{\epsilon}\sigma_1\gamma_{(9)}\left(\psi+\frac{i}{10}\gamma_{(1)}\lambda\right) + \\
& +\frac{40}{9}i\ell e^{-2\phi}\bar{\epsilon}\sigma_3\gamma_{(9)}\left(\psi+\frac{2i}{5}\gamma_{(1)}\lambda\right) - \\
& -12\left(\frac{2}{3}\tilde{D}_{(8)}\delta\tilde{C}_{(2)} + \frac{1}{3}\tilde{B}_{(8)}\delta\tilde{B}_{(2)}\right) + 3\left(\frac{2}{3}\delta\tilde{D}_{(8)}\tilde{C}_{(2)} + \frac{1}{3}\delta\tilde{B}_{(8)}\tilde{B}_{(2)}\right) - \\
& -\frac{63}{4}\left(\frac{1}{3}\tilde{C}_{(6)}\tilde{B}_{(2)}\delta\tilde{C}_{(2)} + \frac{1}{3}(\tilde{B}-\tilde{C})_{(6)}(\tilde{C}-\tilde{B})_{(2)}\delta\tilde{B}_{(2)}\right) + \\
& +\frac{21}{4}\left(\frac{1}{3}\delta\tilde{C}_{(6)}\tilde{B}_{(2)}\tilde{C}_{(2)} + \frac{1}{3}\delta(\tilde{B}-\tilde{C})_{(6)}(\tilde{C}-\tilde{B})_{(2)}\tilde{B}_{(2)}\right) - \\
& -210\tilde{C}_{(4)}\left(\frac{1}{3}\delta\tilde{C}_{(2)}\tilde{B}_{(2)}\tilde{B}_{(2)} + \frac{2}{3}\delta\tilde{B}_{(2)}\tilde{C}_{(2)}\tilde{B}_{(2)}\right) + \\
& +105\delta\tilde{C}_{(4)}\left(\frac{1}{3}\tilde{C}_{(2)}\tilde{B}_{(2)}\tilde{B}_{(2)} + \frac{2}{3}\tilde{B}_{(2)}\tilde{C}_{(2)}\tilde{B}_{(2)}\right) +
\end{aligned}
$$

$$+\frac{315}{16}\frac{i}{3}\left(\delta\tilde{C}_{(2)}\tilde{B}_{(2)}\tilde{B}_{(2)}+2\delta\tilde{B}_{(2)}\tilde{C}_{(2)}\tilde{B}_{(2)}\right)\left(\tilde{C}_{(2)}\delta\tilde{B}_{(2)}-\tilde{B}_{(2)}\delta\tilde{C}_{(2)}\right). \tag{6.73}$$

We will now introduce the standard RR and NS-NS fields, and extend this to the higher rank forms. For this we define:

$$B_{(2)} = \frac{1}{2}\tilde{B}_{(2)}\,, \tag{6.74}$$
$$C_{(0)} = -\frac{1}{2}\ell\,, \tag{6.75}$$
$$C_{(2)} = -\frac{i}{4}\tilde{C}_{(2)}\,, \tag{6.76}$$
$$C_{(4)} = 2\tilde{C}_{(4)} + 3C_{(2)}B_{(2)}\,, \tag{6.77}$$

for which we define curvatures

$$H_{(3)} = 3\partial B_{(2)}\,, \tag{6.78}$$
$$G_{(2n-1)} = (2n-1)\{\partial C_{(2n-2)} - \frac{1}{2}(2n-2)(2n-3)C_{(2n-4)}\partial B_{(2)}\}\,. \tag{6.79}$$

In (6.79) n takes on the values $n = 1, 2, 3$, but this will be extended to $n \leq 6$ below. The corresponding bosonic gauge transformations are

$$\delta B_{(2)} = \partial\Sigma\,, \tag{6.80}$$
$$\delta C_{(2n-2)} = \partial\Lambda_{(2n-3)} + \frac{1}{2}(2n-2)(2n-3)\Lambda_{(2n-5)}\partial B_{(2)}\,. \tag{6.81}$$

We now rewrite the supergravity multiplet in these variables:

$$\delta e_\mu{}^a = 2i\bar{\epsilon}\gamma^a\psi_\mu \tag{6.82}$$
$$\begin{aligned}\delta\psi_\mu &= D_\mu\epsilon - \frac{1}{8}\gamma^{\nu\rho}\,\sigma_3\,\epsilon H_{\mu\nu\rho} - \frac{1}{4}e^\phi(\gamma\cdot G_{(1)})\gamma_\mu\,(i\sigma_2)\,\epsilon - \\ &\quad -\frac{1}{24}e^\phi(\gamma\cdot G_{(3)})\gamma_\mu\,\sigma_1\,\epsilon - \frac{1}{960}e^\phi(\gamma\cdot G_{(5)})\gamma_\mu\,(i\sigma_2)\,\epsilon\,,\end{aligned} \tag{6.83}$$
$$\delta B_{\mu\nu} = 4i\,\bar{\epsilon}\,\sigma_3\,\gamma_{[\mu}\psi_{\nu]}\,, \tag{6.84}$$
$$\delta C_{(0)} = -e^{-\phi}\bar{\epsilon}\,(i\sigma_2)\,\lambda\,, \tag{6.85}$$
$$\delta C_{\mu\nu} = 2ie^{-\phi}\,\bar{\epsilon}\,\sigma_1\,\gamma_{[\mu}\left(\psi_{\nu]} + \frac{i}{2}\gamma_{\nu]}\lambda\right) + C_{(0)}\delta B_{\mu\nu}\,, \tag{6.86}$$
$$\delta C_{\mu\nu\rho\sigma} = 4ie^{-\phi}\,\bar{\epsilon}\,(i\sigma_2)\,\gamma_{[\mu\nu\rho}\left(\psi_{\sigma]} + \frac{i}{4}\gamma_{\sigma]}\lambda\right) + 6C_{[\mu\nu}\delta B_{\rho\sigma]}\,, \tag{6.87}$$
$$\begin{aligned}\delta\lambda &= \frac{i}{2}\gamma^\mu\partial_\mu\phi\,\epsilon - \frac{i}{24}(\gamma\cdot H_{(3)})\,\sigma_3\,\epsilon + \\ &\quad +ie^\phi(\gamma\cdot G_{(1)})\,(i\sigma_2)\,\epsilon + \frac{i}{12}e^\phi(\gamma\cdot G_{(3)})\,\sigma_1\,\epsilon\,,\end{aligned} \tag{6.88}$$
$$\delta\phi = 2\,\bar{\epsilon}\lambda\,. \tag{6.89}$$

The supersymmetry transformations of the RR fields C can be summarized as ($n = 1, 2, 3,\ \mathcal{P}_n = i\sigma_2$ for n even, $\mathcal{P}_n = \sigma_1$ for n odd):

$$\begin{aligned}\delta C_{(2n-2)} &= (2n-2) i e^{-\phi} \bar{\epsilon}\, \mathcal{P}_n\, \gamma_{(2n-3)} \left(\psi_{(1)} + \frac{i}{2n-2}\gamma_{(1)}\lambda\right) + \\ &\quad + \frac{1}{2}(2n-2)(2n-3) C_{(2n-4)} \delta B_{(2)} \,. \end{aligned} \tag{6.90}$$

We will now extend this to the higher-rank forms. We define the following RR fields:

$$C_{(6)} = \frac{1}{4}\tilde{C}_{(6)} + 5 C_{(4)} B_{(2)} \,, \tag{6.91}$$

$$C_{(8)} = \frac{1}{2}\tilde{C}_{(8)} + 7 C_{(6)} B_{(2)} \,, \tag{6.92}$$

$$C_{(10)} = -\frac{3}{4}\tilde{C}_{(10)} + 9 C_{(8)} B_{(2)} \,. \tag{6.93}$$

These combinations transform precisely as (6.90). We have therefore identified the tower of RR forms, in the same form as in [9]. The S-dual of $C_{(10)}$ is however not the field $B_{(10)}$ given in [9]. It turns out that $B_{(10)}$ corresponds precisely to our $\tilde{\mathcal{D}}_{(10)}$.

The S-duals of the $C_{(2n-2)}$ should form a tower of NS-NS forms. If one defines that under S-duality

$$\begin{aligned} C_{(2)} &\to i S_{(2)} \,, \\ C_{(4)} &\to S_{(4)} \,, \\ C_{(6)} &\to -i S_{(6)} \,, \\ C_{(8)} &\to -S_{(8)} \,, \\ C_{(10)} &\to i S_{(10)} \,, \end{aligned} \tag{6.94}$$

then we find

$$S_{(2)} = \frac{i}{4}\tilde{B}_{(2)} \,, \tag{6.95}$$

$$S_{(4)} = 2\tilde{C}_{(4)} + 6 i C_{(2)} S_{(2)} \,, \tag{6.96}$$

$$S_{(6)} = \frac{1}{4}\tilde{B}_{(6)} + 10 i C_{(2)} S_{(4)} \,, \tag{6.97}$$

$$S_{(8)} = \frac{1}{2}\tilde{B}_{(8)} + 14 i C_{(2)} S_{(6)} \,, \tag{6.98}$$

$$S_{(10)} = -\frac{3}{4}\tilde{B}_{(10)} + 18 i C_{(2)} S_{(8)} \,. \tag{6.99}$$

For the case $\ell = 0$ the supersymmetry variations for $S_{(n)}$ are then described by

$$\delta S_{(2n-2)} = (-i)^n (2n-2) e^{-(n-2)\phi} \bar{\epsilon} \mathcal{P}_n \gamma_{(2n-3)} \left(\psi + \frac{n-2}{2n-2}\gamma_{(1)}\lambda\right) + \tag{6.100}$$

$$+ i(2n-2)(2n-3) S_{(2n-4)} \delta C_{(2)} \tag{6.101}$$

where $S_{(0)} = 0$ and $\mathcal{P}_n = \sigma^3$ for n even and $\mathcal{P}_n = i\sigma^2$ for n odd.

potential in quadruplet	potential in doublet	associated brane	tension
$C_{(10)}$		D9-brane	g_S^{-1}
$D_{(10)}$	$\mathcal{D}_{(10)}$	solitonic brane	g_S^{-2}
$E_{(10)}$	$\mathcal{E}_{(10)}$	exotic	g_S^{-3}
$B_{(10)}$		exotic	g_S^{-4}

Table 1: Ten-form potentials in string frame, the corresponding branes and their tension in terms of the string coupling g_S.

7. Summary and discussion

In this work we showed that the standard formulation of IIB supergravity can be extended to include a doublet and a quadruplet of ten-form potentials. We argued that no other independent ten-forms can be added to the algebra. We have been using a "SU(1,1)-democratic" formulation, in which *all* forms are described together with their magnetic duals. Furthermore, all forms transform in a given representation under the duality group $\mathrm{SL}(2,\mathbb{R})$. The previously known RR-ten-form potential $C_{(10)}$ is contained in the quadruplet. The other previously known ten-form (named $B_{(10)}$ in [9]) is in the doublet and hence not S-dual to $C_{(10)}$ [13].

We have shown that all ten-form potentials have a leading term

$$\delta X_{(10)} \sim e^{n\phi}\bar{\epsilon}\gamma_{(9)}\psi \qquad \text{at } l=0 \tag{7.1}$$

in their supersymmetry transformation in string frame where $X_{(10)}$ represents a generic ten-form potential.

Such ten-form potentials naturally occur as the leading contribution in Wess-Zumino terms for space-time filling branes with tension g_S^n. The resulting branes can be found in table 1. These branes and their relevance for theories with sixteen supercharges will be discussed in some detail in a forthcoming paper [17].

It would be interesting to see how these findings are compatible with the known S-duality relations between the Heterotic and type-I superstrings. It is well-known that the (Nambu-Goto part of the) tree-level action of the type-I (Heterotic) superstring scales with g_S^{-1} (g_S^{-4}) [22]. The interpretation of the g_S^{-4} term at the Heterotic side is not clear. However, the results presented in table 3, appendix B, and the S-duality assignments of the ten-forms (6.44) open up the possibility to extend this, consistent with S-duality, to the scaling behaviour $g_S^{-1}+g_S^{-3}$ for the type-I superstring and $g_S^{-2}+g_S^{-4}$ for the Heterotic superstring such that the Nambu-Goto term at the Heterotic side contains the more conventional g_S^{-2} behaviour.

Work on the relation of string- and M-theory with the Kac-Moody algebras E_{11} [23, 24] and E_{10} [25, 26] has an interesting connection with our results. In [27] it was pointed out that E_{10} and E_{11} give rise to different IIB ten-form potentials. In particular, E_{10} does not give rise to ten-forms, whereas E_{11} supports a doublet and a quadruplet of ten-forms [28]. The latter is in agreement with our results.

It will be worthwile to derive the superspace formulation of our results. Note that, although ten-form potentials have identically zero field-strengths, this is not true for the ten-form superpotentials. It would be interesting to calculate the eleven-form curvatures in flat superspace and to see to which kind of Wess-Zumino terms they give rise to. This is the first step towards the construction of a kappa-symmetric Green-Schwarz action for *all* 9-branes.

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A. Conventions

The Levi-Civita symbol used in this paper is a tensor, and therefore includes the appropriate powers of det e.

Some useful properties of the complex fermions are:

$$\psi_\mu = -\gamma_{11}\psi_\mu\,, \tag{A.1}$$

$$\lambda = \gamma_{11}\lambda\,, \tag{A.2}$$

$$D_\mu\epsilon = \left(\partial_\mu + \frac{1}{4}\omega_\mu{}^{ab}\gamma_{ab} - \frac{i}{2}Q_\mu\right)\epsilon\,, \tag{A.3}$$

$$(\bar\chi_1\gamma^{\mu_1\cdots\mu_n}\chi_2)^* = \bar\chi_2\gamma^{\mu_n\cdots\mu_1}\chi_1 = (-1)^n\bar\chi_{1C}\gamma^{\mu_1\cdots\mu_n}\chi_{2C}\,, \tag{A.4}$$

$$\bar\chi_1\gamma^{\mu_1\cdots\mu_n}\chi_2 = (-1)^{n(n+1)/2}\bar\chi_{2C}\gamma^{\mu_1\cdots\mu_n}\chi_{1C}\,. \tag{A.5}$$

In these equations χ_i are arbitrary spinors, not necessarily Majorana or Weyl.

For the duality transformations of γ-matrices we have:

$$\gamma^{\mu_1\cdots\mu_n} = -(-1)^{\frac{1}{2}n(n-1)}\frac{1}{(10-n)!}\epsilon^{\mu_1\cdots\mu_{10}}\gamma_{\mu_{n+1}\cdots\mu_{10}}\gamma_{11}\,. \tag{A.6}$$

Table 2 gathers the values of the U(1) weights of the different fields. The zehnbein $e_\mu{}^a$ and all form-fields $A_{(2n)}$ have weight zero.

V_+^α	1	$G_{(3)}$	1	$A_{(n)}$	0		
V_-^α	-1	$G_{(7)}$	1				
ϵ	$\frac{1}{2}$	ϵ_C	$-\frac{1}{2}$	$\bar\epsilon$	$-\frac{1}{2}$	$\bar\epsilon_C$	$\frac{1}{2}$
ψ_μ	$\frac{1}{2}$	$\psi_{C\,\mu}$	$-\frac{1}{2}$	$\bar\psi_\mu$	$-\frac{1}{2}$	$\bar\psi_{C\mu}$	$+\frac{1}{2}$
λ	$\frac{3}{2}$	λ_C	$-\frac{3}{2}$	$\bar\lambda$	$-\frac{3}{2}$	$\bar\lambda_C$	$\frac{3}{2}$

Table 2: Table of U(1) weights.

B. Truncations

We briefly sketch how to apply the heterotic and type-I truncations [9] to our IIB results and give a list of the fields surviving the truncation.

We first express the complex spinor ϵ in terms of two real spinors

$$\epsilon = \epsilon_1 + i\epsilon_2 \,. \tag{B.1}$$

The heterotic truncation is then given by setting

$$\epsilon = \pm\epsilon_C \,. \tag{B.2}$$

We will work with the "+" choice. We also need to make a choice of gauge for the scalars. We make the same choice as in section 6:

$$V_+^2 = V_-^1 \,. \tag{B.3}$$

Plugging (B.2) into the SUSY variation of ψ we find

$$\psi = \psi_C \,. \tag{B.4}$$

Similarly, we use the SUSY variations of the other fields to find how the truncation acts on all the fields

$$\begin{aligned}
\psi &= \psi_C \,, \qquad \lambda = \lambda_C \,, && \text{(B.5)}\\
V_+^2 &= V_-^1 \,, \qquad V_-^2 = V_+^1 \,, && \text{(B.6)}\\
A_{(2)}^1 &= A_{(2)}^2 \,, && \text{(B.7)}\\
A_{(4)} &= 0 \,, && \text{(B.8)}\\
A_{(6)}^1 &= -A_{(6)}^2 \,, && \text{(B.9)}\\
A_{(8)}^{11} &= -A_{(8)}^{22} \,, \qquad A_{(8)}^{12} = 0 \,, && \text{(B.10)}\\
A_{(10)}^1 &= A_{(10)}^2 \,, && \text{(B.11)}\\
A_{(10)}^{111} &= -A_{(10)}^{222} \,, \qquad A_{(10)}^{112} = -A_{(10)}^{122} \,. && \text{(B.12)}
\end{aligned}$$

We also observe that the relations for the scalars (B.6) imply, using the reality properties of the scalars and (2.16), that $z = \bar{z}$. This implies that the axion is eliminated by the truncation, using (2.20) and (2.24).

The type-I truncation is given by setting

$$\epsilon = \pm i\epsilon_C \tag{B.13}$$

where we work with the "+"-choice again. We choose $V_+^2 = V_-^1$ again and find from the SUSY variations

$$\begin{aligned}
\epsilon &= i\epsilon_C \,, \qquad \psi = i\psi_C \,, \qquad \lambda = -i\lambda_C \,, && \text{(B.14)}\\
V_+^2 &= V_-^1 \,, \qquad V_-^2 = V_+^1 && \text{(B.15)}
\end{aligned}$$

type-I truncation	heterotic truncation
ϕ	ϕ
$\tilde{C}_{(2)} \sim e^{-\phi}$	$\tilde{B}_{(2)} \sim e^{0\phi}$
$\tilde{C}_{(6)} \sim e^{-\phi}$	$\tilde{B}_{(6)} \sim e^{-2\phi}$
$\tilde{D}_{(8)} \sim e^{-2\phi}$	$\tilde{D}_{(8)} \sim e^{-2\phi}$
$\tilde{\mathcal{E}}_{(10)} \sim e^{-3\phi}$	$\tilde{\mathcal{D}}_{(10)} \sim e^{-2\phi}$
$\tilde{C}_{(10)} \sim e^{-\phi}$	$\tilde{B}_{(10)} \sim e^{-4\phi}$
$\tilde{E}_{(10)} \sim e^{-3\phi}$	$\tilde{D}_{(10)} \sim e^{-2\phi}$

Table 3: Field contents of the type-I and heterotic truncations. After the tilde we indicate how the field scales with respect to the dilaton. The entries in every line are S-dual to each other (up to a factor).

$$A^{1}_{(2)} = -A^{2}_{(2)}\,, \tag{B.16}$$
$$A_{(4)} = 0\,, \tag{B.17}$$
$$A^{1}_{(6)} = A^{2}_{(6)}\,, \tag{B.18}$$
$$A^{11}_{(8)} = -A^{22}_{(8)}\,, \qquad A^{12}_{(8)} = 0\,, \tag{B.19}$$
$$A^{1}_{(10)} = -A^{2}_{(10)}\,, \tag{B.20}$$
$$A^{111}_{(10)} = A^{222}_{(10)}\,, \qquad A^{112}_{(10)} = A^{122}_{(10)}\,. \tag{B.21}$$

As in the case of the heterotic truncation, the axion is eliminated by the truncation. We collect the surviving fields of both truncations in table 3.

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